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(54) METHOD AND SYSTEM FOR MARKET BASED RESOURCE ALLOCATION

Related U.S. Application Data

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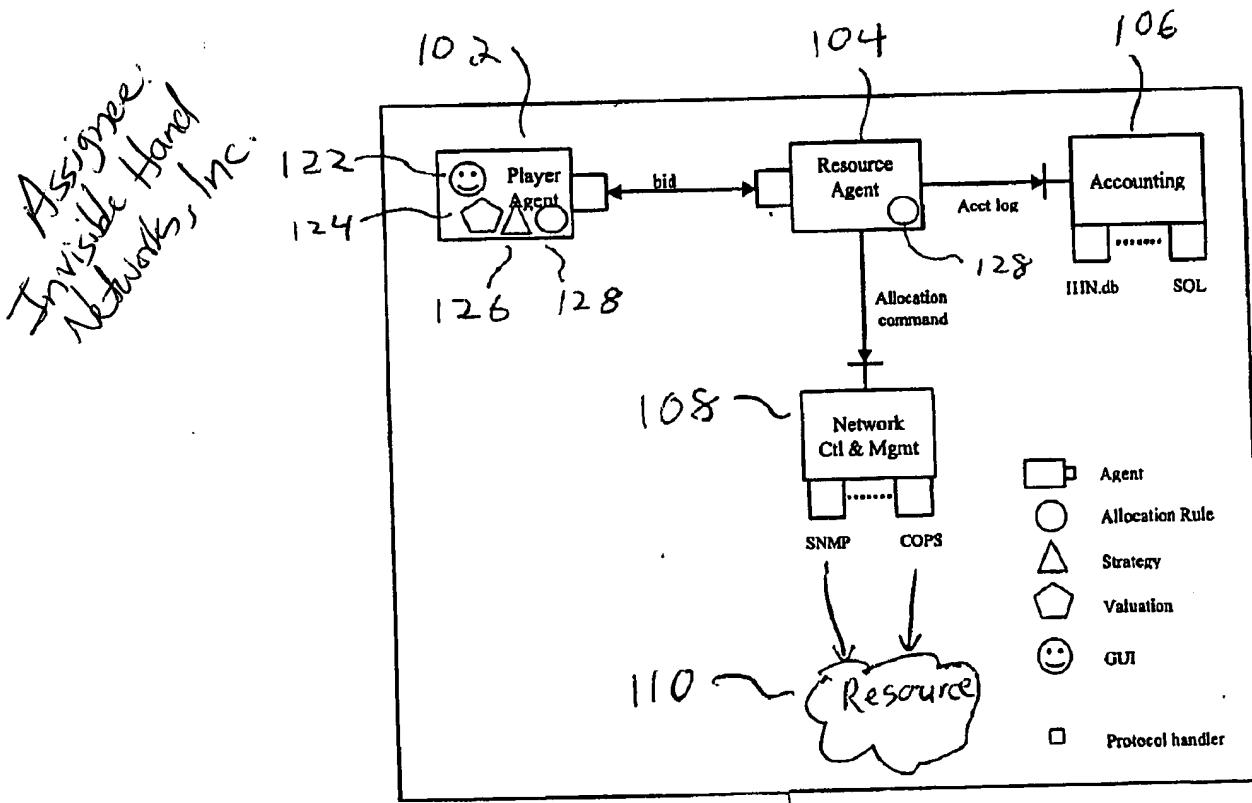
(21) Appl. No.: 09/854,325

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(22) Filed: May 12, 2001

ABSTRACT

The present invention provides a platform for resource allocation in real-time. One or more software resource agents interact with software player agents, which are one or more of buyers and seller agents, to reach an agreement on a price for a specific resource.



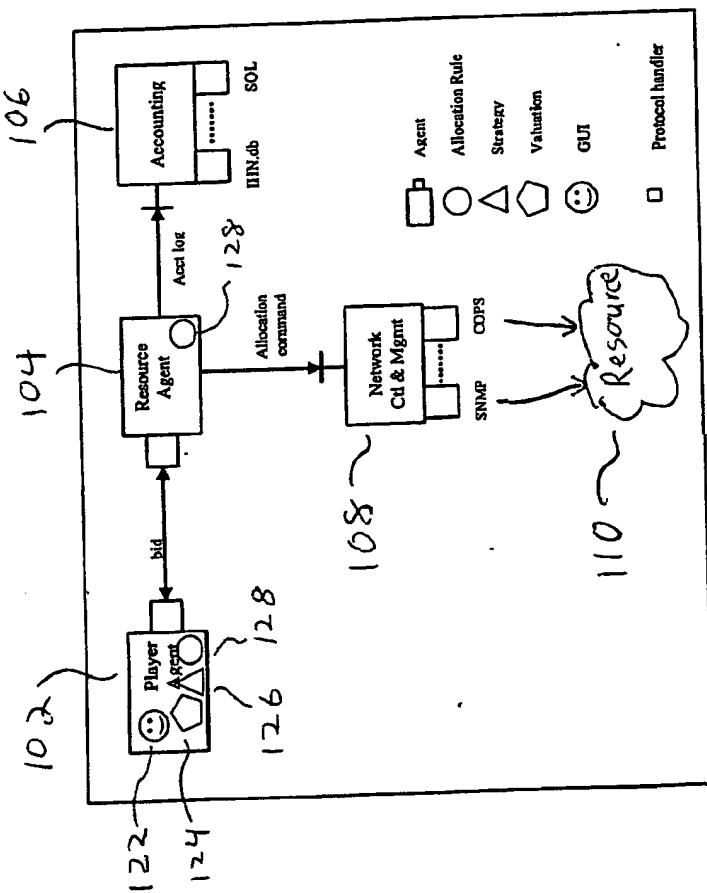


Fig 1

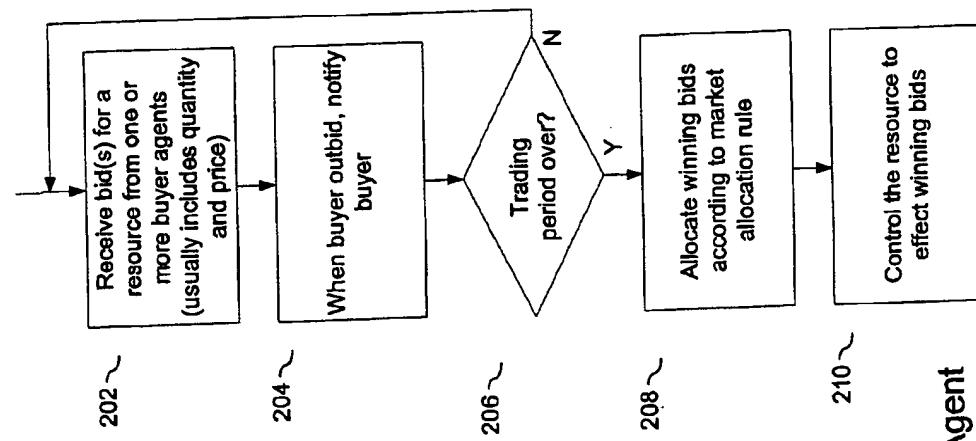
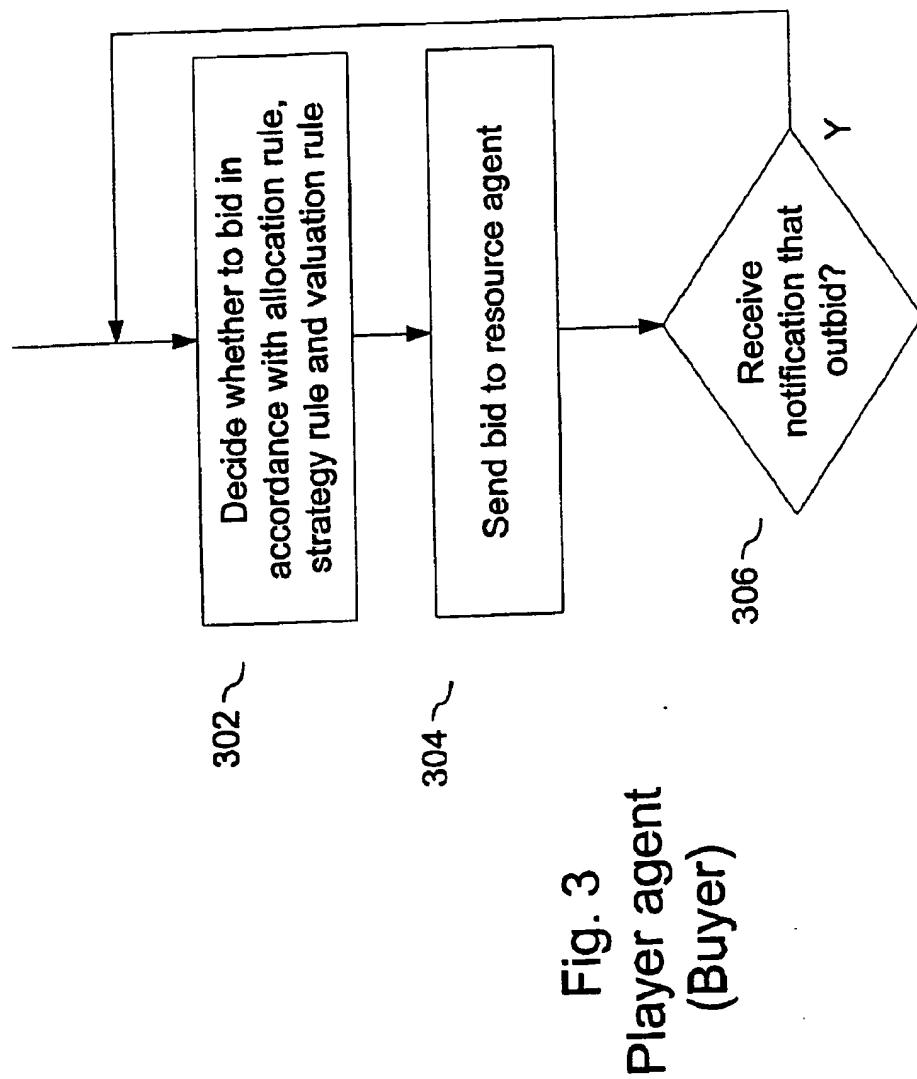


Fig. 2  
Resource Agent



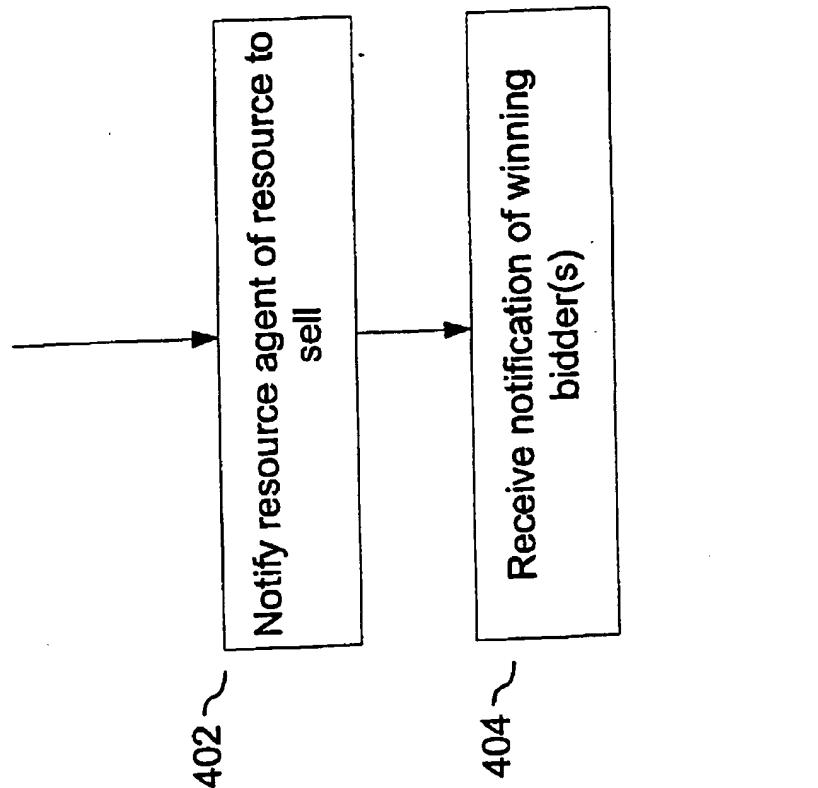


Fig. 4  
Player agent  
(Seller)

100 units of resource:  
A bids for 50 at \$3,  
B bids for 30 at \$2  
C bids for 30 at \$1  
D bid for 20 at \$0.50

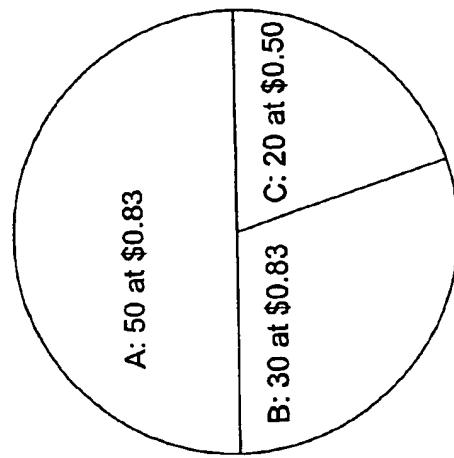


Fig. 5  
Example Market Allocation Rule  
(PSP)

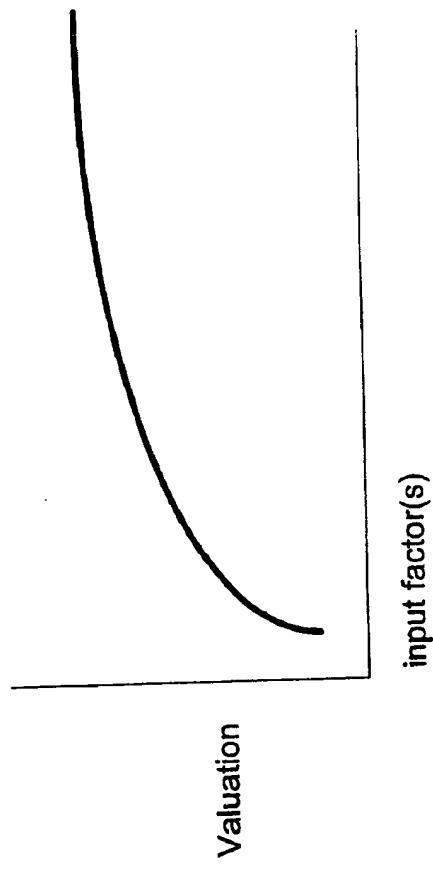


Fig. 6(b)  
Example  
Valuation Rule

Quantity	Price	Valuation
-	-	-
-	-	-
-	-	-
-	-	-
-	-	-

Fig. 6(a)  
Example  
Valuation Rule

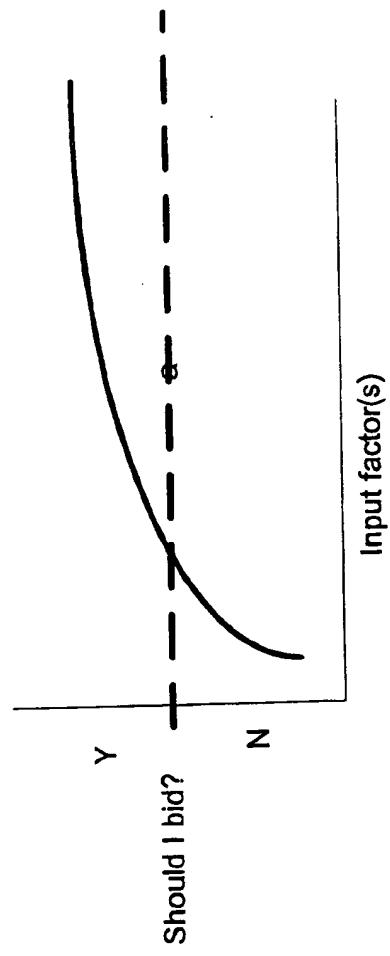


Fig. 7(a)  
Example Strategy  
Rule

If allocation rule is PSP  
[actions for PSP bidding]  
If allocation rule is English auction  
[actions for English auction]

Fig. 7(b)  
Example Strategy  
Rule

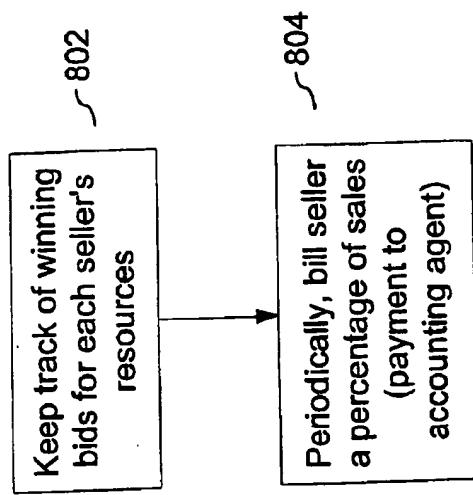


Fig. 8

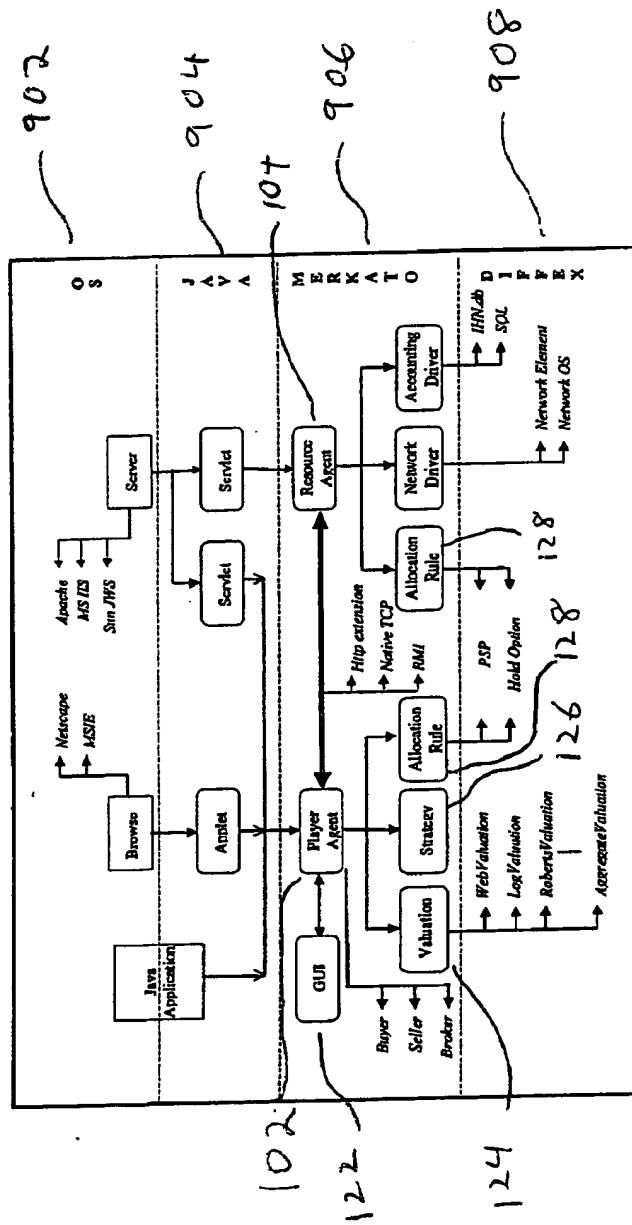


Fig. 9(a)

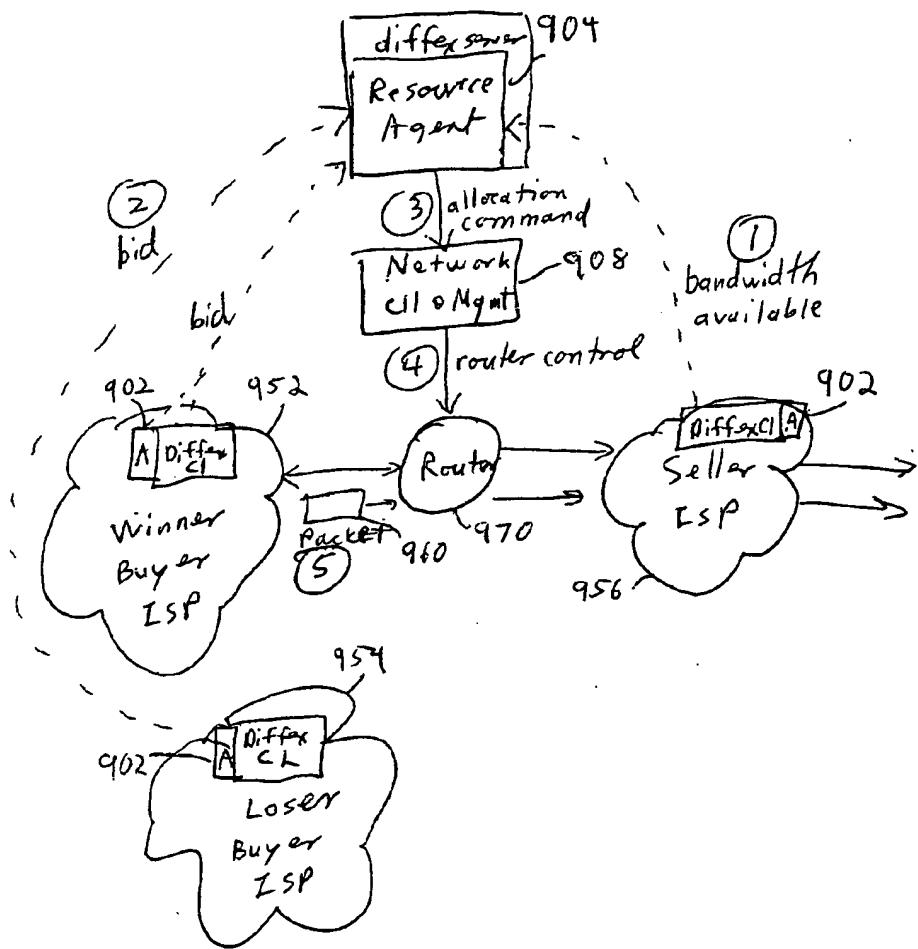


Fig 9(b)

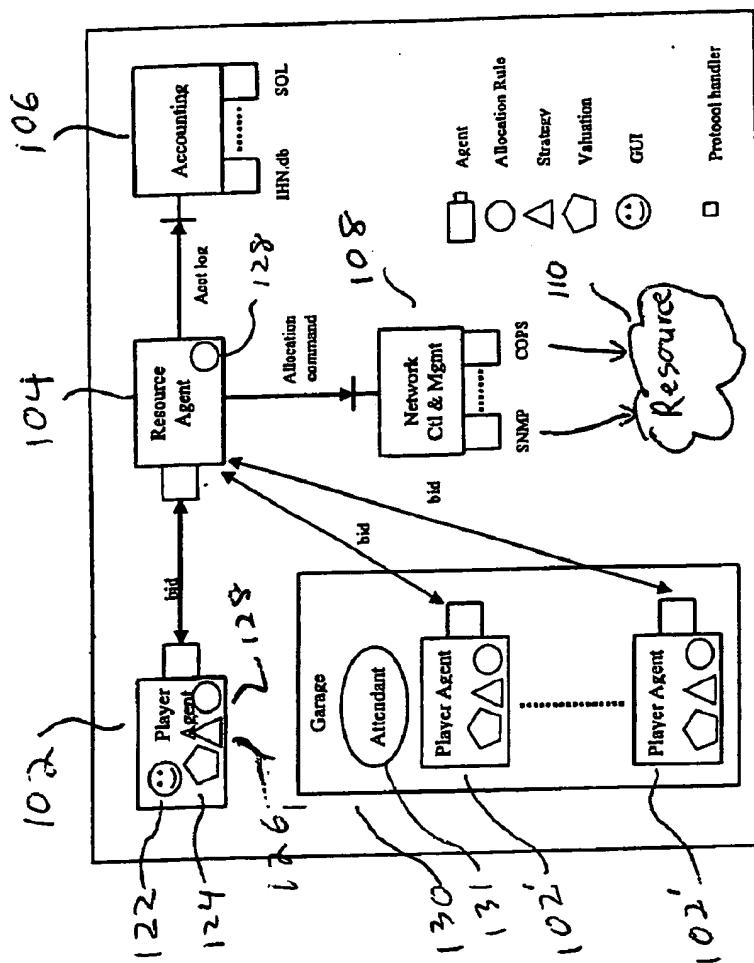


Fig 10

```

<?xml version="1.0" encoding="UTF-8" ?>
- <AuctionPlayer context="http://HOSTNAME:HTTP_PORT/bx/garage">
  - <SingleFrameGUI>
    <TextPanel name="News" height="50" visible="true" border="false" />
    <LoginPanel name="Login" height="160" visible="true" border="true" />
    <ResourceAgentPanel name="ResourceAgent" height="80" visible="true"
      border="true" />
    <UploadAgentPanel name="Garage" height="80" visible="true"
      border="true" />
    <BidCanvasPanel name="BidCanvas" height="180" visible="false"
      border="true" />
  - <StrategyChoicePanel name="Strategies" height="160" visible="true"
    border="true">
    <StrategyPanel name="Manual" strategy="ManualStrategy" />
    <StrategyPanelNotEditable name="Auto" strategy="TruthfulStrategy" />
  />/StrategyChoicePanel>
  - <ValuationChoicePanel name="Valuations" height="240" visible="false"
    border="true">
    <WebValuationPanel name="Web Valuation" valuation="WebValuation" />
    <ValuationPanel name="Elastic Demand" valuation="RobertsValuation" />
    <ValuationPanel name="Inelastic Demand" valuation="LinearValuation" />
    <BudgetValuationPanel name="Budget Valuation" label=""
      valuation="BudgetValuation" />
  />/ValuationCholcePanel>
  <PlayerInfoPanel name="Allocation" height="120" visible="true"
    border="true" />
  <BudgetPanel name="Budget" height="80" visible="false" border="true" />
  <DisplayPanel name="Units" height="80" visible="false" border="true" />
  <IPAddressPanel name="IP" height="110" visible="false" border="true" />
  <ConnectionPanel name="Connection" height="140" visible="false"
    border="true" />
  <BidTablePanel name="Bid Table" height="400" visible="false"
    border="true" />
  <BidGraphPanel name="Bid Graph" height="400" visible="false"
    border="true" />
  <AllocationGraphPanel name="Allocation Graph" height="400" visible="false"
    border="true" />
</SingleFrameGUI>
<PlayerIdentity name="USERNAME" passwd="PASSWD" ipaddress="IP_ADDRESS"
  netmask="NETMASK" />
- <LinearValuation label="Inelastic Demand">
  <Parameter name="qmax" value="45000.0" label="Kbps" />
  <Parameter name="vmax" value="44928.0" label="$/month" />
</LinearValuation>
- <RobertsValuation current="false" label="Elastic Demand">
  <Parameter name="qmax" value="45000.0" label="Kbps" />
  <Parameter name="vmax" value="4928.0" label="$/month" />
</RobertsValuation>
- <BudgetValuation current="true" label="Budget Valuation">
  <Parameter name="qmax" value="1000.0" label="Kbps" />
  <Parameter name="budget" value="100.0" label="$/month" />
</BudgetValuation>

```

(a)

```
- <WebValuation label="Web Valuation">
  <param name="delay" value="100.0" />
  <param name="hitspermonth" value="100000.0" />
  <param name="filesize" value="1000.0" />
  <param name="centsperhit" value="0.1" />
  <param name="randomize" value="false" />
</WebValuation>
<Parameter name="budget" value="51840.6" label="$/month" />
<ManualStrategy current="false" label="Manual" />
<TruthfulStrategy current="true" label="Auto" />
<resourceAgentURL nickname="RESOURCE_NAME"
  current="true">http://HOSTNAME:HTTP_PORT/bx/RESOURCE_NAME</resourceAge
<uploadURL nickname="HOSTNAME
  garage">http://HOSTNAME:HTTP_PORT/bx/garage</uploadURL>
<param name="playerInterval" value="2000" />
<param name="timeout" value="2000" />
<param name="timelabel" value="min" />
<param name="currencylabel" value="c" />
<param name="quantitylabel" value="Mbps" />
<param name="debug" value="false" />
</AuctionPlayer>
```

II (b)

```

/*
 * File:          Truthful.java
 *
 * Remark:        Strategy for player with diminishing returns
 *
 * $Id: Truthful.java,v 1.16      07:43:19 cobe Exp $
 *
 */
package ihm.merkato;
import org.w3c.dom.*;
import com.sun.xml.tree.XmlDocument;

/**
 * The strategy that bids the truthful best reply as in Proposition 1 o
f
 * the PSP paper.
 * It will only submit the bid if utility will be increased by at least
 * epsilon.
 *<p>
 * @author Nemo Semret
 *{
 */
public class Truthful extends AuctionStrategy {

    Bid tmp = createBid();
    /**
     * Finds truthful best reply as in Proposition 1 of
     * the PSP paper.
     * Sets the bid at the player if utility will be increased by at leas
t
     * epsilon.
     *<p>
     * If timelogging is enabled, this will write to the player's
     * log a line with current time, bid, allocation, and utility,
     * at each call.
     * @see #epsilon
     * @see ihm.merkato.AuctionPlayer#setBid
    */
    public boolean bid() {

        double lq=0, uq= getPlayer().getValuation().qmax(), mq= (uq+lq)/2,
               dq = getPlayer().dq();

        if(debug()) {
            getPlayer().log("q range = ["+lq+
                            ","+uq+"] dq="+dq+
                            " Q="+getPlayer().stuff());
            getPlayer().addnews(".");
        }
    }
}

```

(2(a))

```

// see Proposition 1
int i=0;
double mp, dv;
while( uq-lq >dq && i<20) {
    i++;
    mq = (lq+uq)/2;
/*
    if(mq < getPlayer().stuff() -
        (getBidder().getBidList()).demandAtPrice(
        getPlayer().dval(mq, mq+dq),
        getPlayer().getId()))
    */
    // the following is equivalent and more general
    dv=getPlayer().getValuation().dval(mq, mq+dq);
    mp=getBidder().getBidList().marketPrice(getPlayer().stuff()
                                              -mq,
                                              getPlayer().getId())
};

if(debug())
    getPlayer().log("i="+i+" mq="+mq+" dv="+dv+" mp="+mp);

if(dv>mp)
    lq=mq;
else
    uq=mq;
}

tmp.bidderid = getPlayer().getId();
tmp.price = Data.MAXPRICE;
tmp.qty = lq;

if(debug())
    getPlayer().log(""+i+" steps. q range = ["+lq
                  +", "+uq+"] currentbid="+
                  getBidder().anteBid()+" found "+tmp);

if(util(tmp) <0) {
    uq= tmp.qty;
    lq=0;
}

i=0;
while(uq-lq>dq && i<20) {
    tmp.qty = (uq+lq)/2;
    i++;
    if(debug())
        getPlayer().log("i="+i+" q="+tmp.qty);
    if(util(tmp) <0)
        uq= tmp.qty;
    else
        lq = tmp.qty;
}

```

12(b)

```

        }

        // need this in case the above loop is just outside the budget
        while (util( tmp ) <0 && tmp.qty>0 && i <40) {
            i++;
            tmp.qty -=dq;
            if(debug())
                getPlayer().log("i="+i+" q="+tmp.qty);
        }

        if(debug())
            getPlayer().log(""+i+" steps. range=[ "+lq+", "+uq+" ] currentbid="+
                getBidder().anteBid().qty);

| 232 tmp.price = getPlayer().getValuation().dval(tmp.qty, tmp.qty+dq);

        double u = getPlayer().currentUtil();
        double newu = util(tmp);

        if(debug()) {
            getPlayer().log("currentalloc="+
                getBidder().currentAllocation()+
                " newbid="+tmp+" ante bid="+
                getBidder().anteBid());
            getPlayer().log("u=" +u+ " newu=" +newu+ " antes="
                +util(getBidder().anteBid())+
                +" fee=" +getBidder().bidFee()+
                +" epsilon=" +epsilon()+
                +" avgdur=" +getAvgDuration());
        }

        if(getPlayer().trace()) {
            Bid alloc = getBidder().currentAllocation();

            getPlayer().log(""+getBidder().anteBid().qty
                +"\t"+getBidder().anteBid().price+"\t"+
                alloc.qty+"\t"+alloc.price+"\t"+
                getPlayer().currentUtil());
        }

| 234 { if ( newu > u + epsilon() ) {
            if(debug()) getPlayer().addnews("*");
            return getBidder().setBid(tmp.qty, tmp.price);
        } else {
            if(debug()) getPlayer().addnews("-");
            return false;
        }
    }
}

```

| 2(c)

```
/*
 *      BidList object
 *
 * File:          PSPBidList.java
 *
 *
 *
 *
 *
 *
 *
 */
package ihm.diffpex;

import ihm.merkato.Bid;
import ihm.merkato.Data;
/**
 *
 */
public class PSPBidList extends ihm.merkato.GenericBidList {

    /**
     * Compute an allocation given the current profile of opponents
     * in
     * this bidlist. This class uses the Progressive-Second-Price
     * auction rule.
     * @param tb The bid for which the allocation is to be calculate
     * d.
     * @param Q  The total quantity of resource available.
     */
    public Bid allocation(Bid tb, double Q) {
        return PSPallocation(tb, Q);
    }

    /**
     * Compute an allocation given the current profile of opponents
     * in
     * this bidlist, with the Progressive-Second-Price
     * auction rule.
     * @param tb The bid for which the allocation is to be calculate
     */

}
```

13(a)

```

d.
 * @param Q  The total quantity of resource available.
 */

private synchronized Bid PSPAllocation(Bid tb, double Q) {
    Bid index = top;
    Bid alloc= new Bid();
    double leftover = Q; //leftover with player id
    double leftoverwo =Q; //leftover without player id
    double qAj,qAj0, num=0, den=0;
    boolean gotcha = false;

    alloc.qty = Math.min(tb.qty,
                         Math.max(Q-demandAtPrice(tb.price, tb.bid
derid),0));

    while(index.next != null) {
        index = index.next;

        if(index.bidderid != tb.bidderid) {

            if(index.price <= tb.price && !gotcha) {
                leftover -= tb.qty;
                if(leftover <=0)leftover=0;
                gotcha = true;
            }

            qAj = (index.qty <= leftover ? index.qty : leftover);
            qAj0= (index.qty <= leftoverwo ? index.qty : leftoverwo);
            num += (index.price* (qAj0- qAj));
            //      den += (qAj0- qAj);

            leftoverwo -= index.qty;
            leftover -= index.qty;
            if(leftover <=0)leftover=0;
            if(leftoverwo <=0)leftoverwo=0;

        }
    }

    //      if (!gotcha) alloc.qty = (tb.qty <= leftover ? tb.qty :

```

(3 (b))

```
leftover);

//    alloc.price = den>0 ? num/den : 0;
alloc.price = alloc.qty>0 ? num/alloc.qty : 0;
alloc.bidderid = tb.bidderid;

return alloc;
}

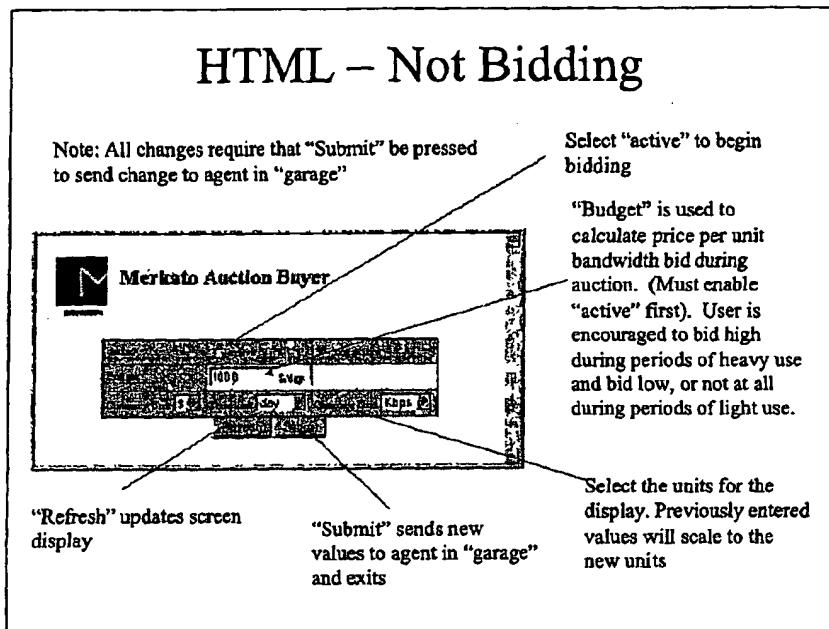
<**
 * Bids with ID#0 are not counted.
 */
public double revenue(double Q) {
    Bid index = top;
    double r=0;
    int I=0;
    Bid al;
    while (index.next != null) {
        index = index.next;
        I++;
        if(index.bidderid!=0) {
            al = allocation(index,Q);
            r+= al.qty*al.price;
            //value(index.bidderid,Q);
        }
    }
    return r;
//    if(I==0) return 0;
//    else return r -= (I-1)*value(Data.NOBODY, Q);
}

}
// substituted 8 float to double
```

13(c)

```
<?xml version="1.0" encoding="UTF-8" ?>
- <GenericAuctionAgent
  context="http://HOSTNAME:HTTP_PORT/bx/RESOURCE_NAME">
  <PlayerIdentity name="RESOURCE_USER" passwd="RESOURCE_PASSWD"
    ipaddress="127.0.0.1" netmask="255.255.255.255" />
- <PSPBidList>
  <param name="randomduration" value="false" />
  <param name="duration" value="60000" />
  <param name="mustconv" value="true" />
  <param name="bidfee" value="0.01" />
  <param name="capacity" value="20000.0" />
</PSPBidList>
<UnixCryptAuthenticator passwdfile="MERKATO_HOME/accounts/passwd" />
- <LinearValuation>
  <Parameter name="qmax" value="QMAX_VAL" label="QMAX_UNITS" />
  <Parameter name="vmax" value="VMAX_VAL" label="VMAX_UNITS" />
</LinearValuation>
<param name="accountingDriverClass"
  value="ihn.merkato.AccountManager" />
<param name="accountFile"
  value="http://HOSTNAME:HTTP_PORT/bx/dbstub" />
<param name="hwDriverClass" value="RESOURCE_DRIVER_CLASS" />
<param name="hwDevice" value="RESOURCE_DRIVER_INIT" />
<param name="maxNBids" value="100" />
<param name="verbose" value="true" />
<param name="rememberIds" value="false" />
<param name="clientTimeout" value="60000" />
<param name="serverTimeout" value="30000" />
<param name="pause" value="5000" />
<param name="detailedlog" value="true" />
<Parameter name="maxBidFee" value="1.0" label="$" />
<Parameter name="maxAccountBalance" value="10000.0" label="$" />
</GenericAuctionAgent>
```

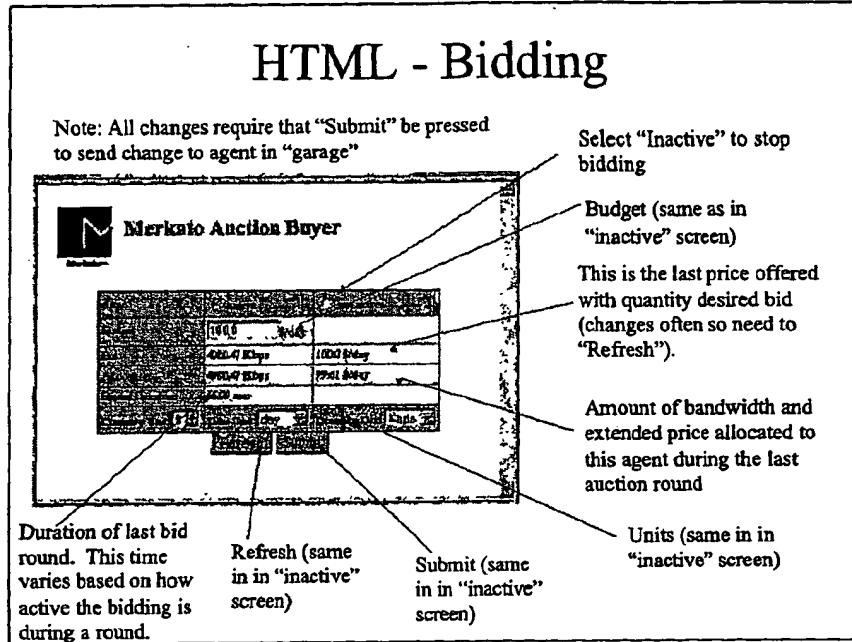
Fig 14



There is no cost accrued to buyers who are not bidding. They will be placed in the best-effort queue until they elect to bid for bandwidth.

"Submit" updates the budget value of the garaged agent to what you have entered into this screen and exits. At this point, it exits to a generic Merkato screen, but for customers, it will exit to a StreamingHand page.

Fig 15(a)

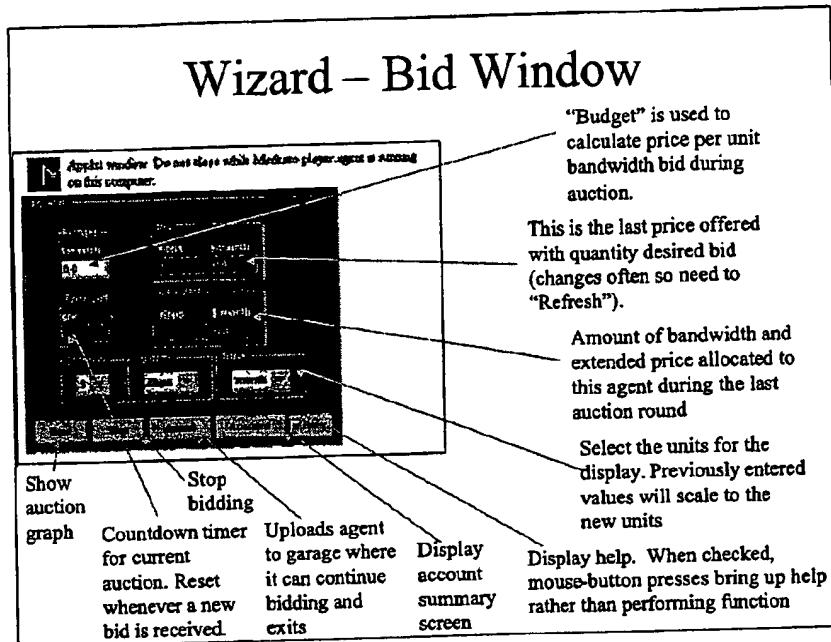


User will generally want to bid high during period of heavy use and lower during periods of light use.

The agent will attempt to obtain as much bandwidth as possible without exceeding budget. Conversely, the agent will request smaller and smaller amounts of bandwidth until they can obtain something for the budget price.

"Refresh" updates the screen. It does not send any changes to the "budget" value to the garaged agent. "Submit" does this (and then exits).

Fig 15(b)



"Stop" means to stop bidding. This bidding agent will not be charged and they will be placed in the shared best-effort queue.

"Upload" uploads the configuration to the garaged agent. Note that this will change some advanced settings to those assumed by this simple valuation and strategy model.

This simple budget-based valuation model has the bidding agent attempt to get as much bandwidth as possible without exceeding the budget number.

The strategy is based on the formula:  $\text{price-per-unit-bandwidth} * \text{bandwidth-allocated} = \text{total-price-paid}$

Where the total-price-paid ("budget") is held constant, and the other two variables allowed to be altered.

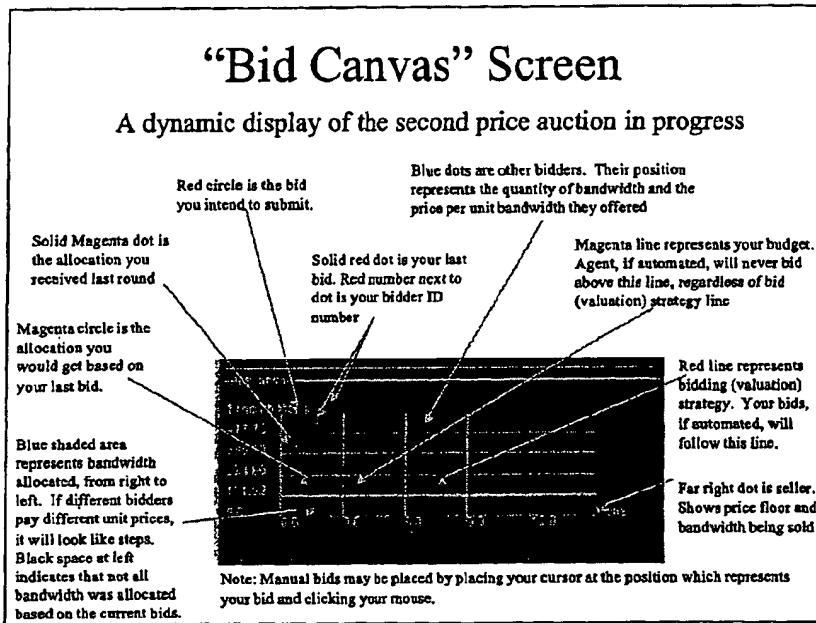
Following this strategy, the bidder will first attempt to get all the bandwidth the seller is offering for their budgeted amount, which works out to the lowest possible price-per-unit-bandwidth. If unsuccessful, the bidding agent gradually increases the offered price-per-unit-bandwidth and decreases the desired amount of bandwidth, until they successfully win an allocation.

If all bidders follow this valuation model, they will each get a bandwidth allocation that is the same proportion to total bandwidth as their budget is to the combined budgets of all bidders.

From the "Help" screen"

- Press **Start** to tell your agent to start bidding for you.
- Press **Stop** to tell your agent to stop.

Fig 15(c)



Red Valuation Line and Magenta Budget line are superimposed when “Budget” valuation is being used (default for “HTML” and “Wizard” Agent Interfaces).

Often the Red circle, red dot and magenta circle will be very close together.

Fig 15(d)

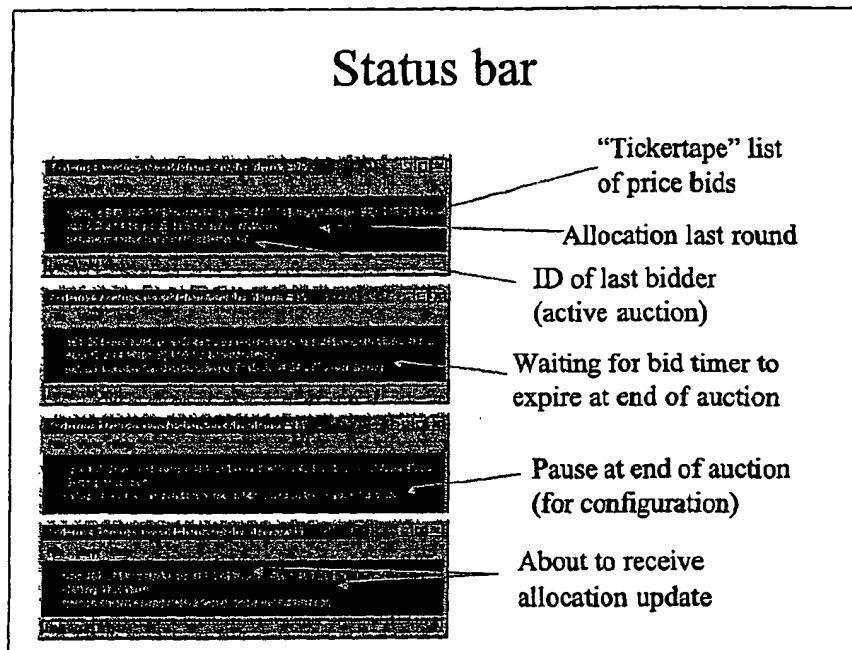


Fig 15(e)

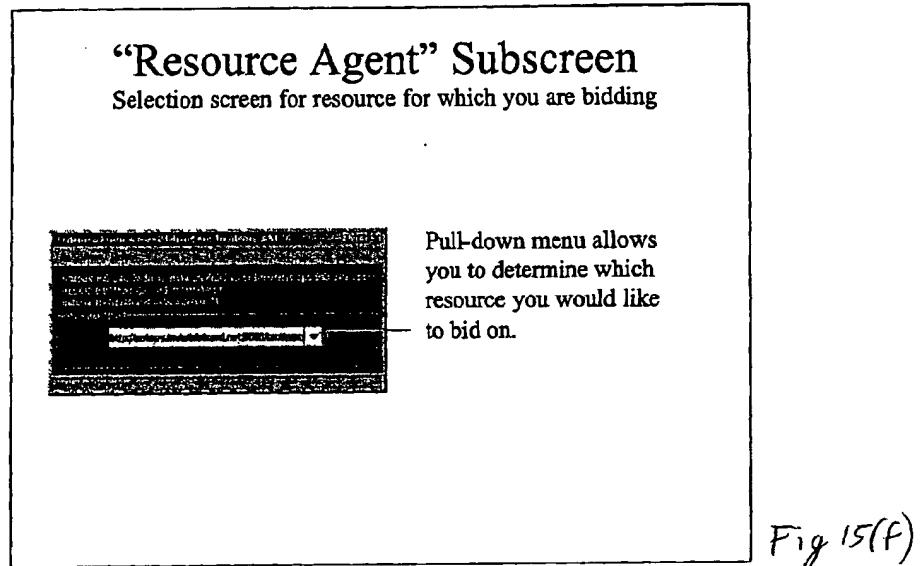
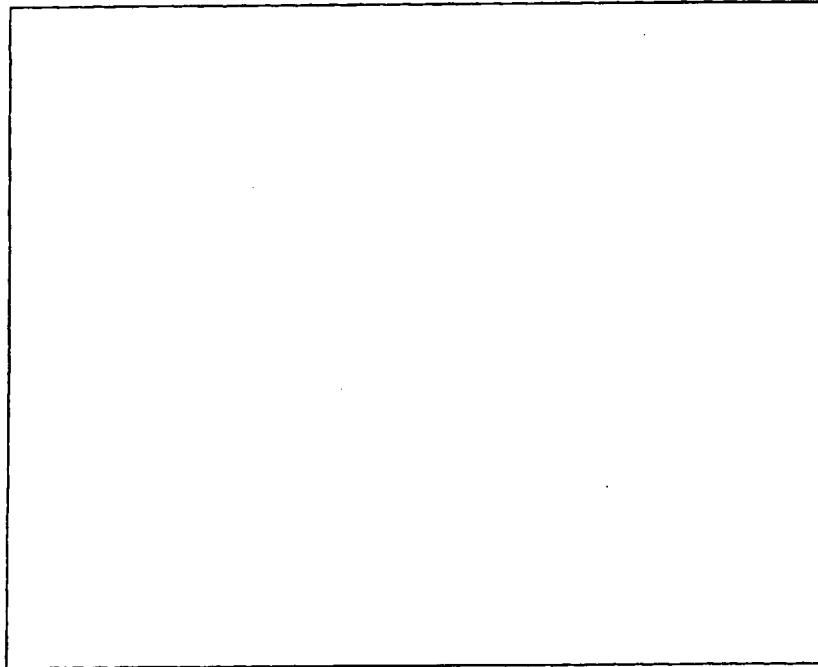


Fig 15(f)



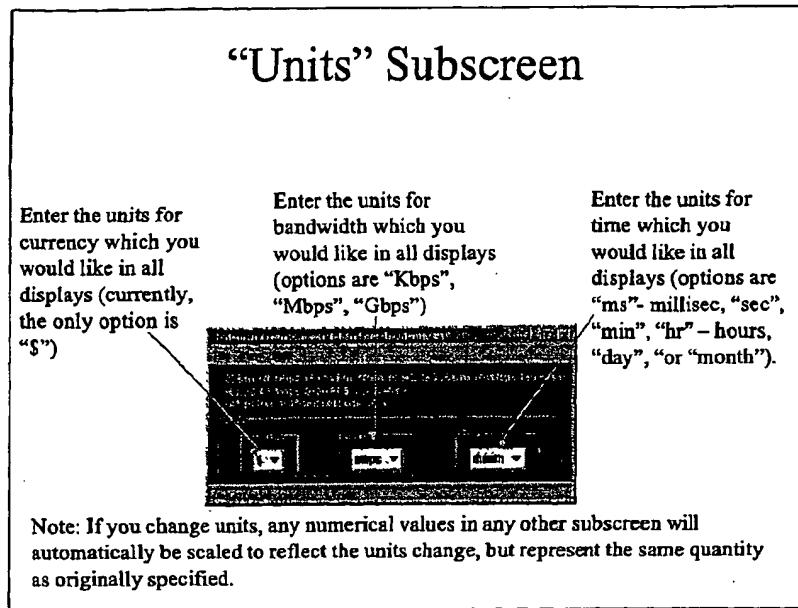


Fig 15(g)

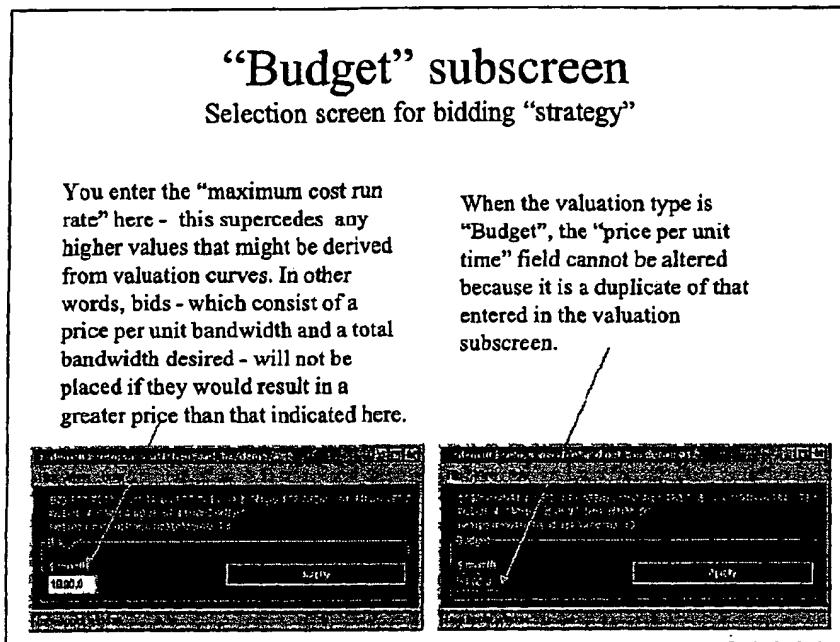


Fig 15(h)

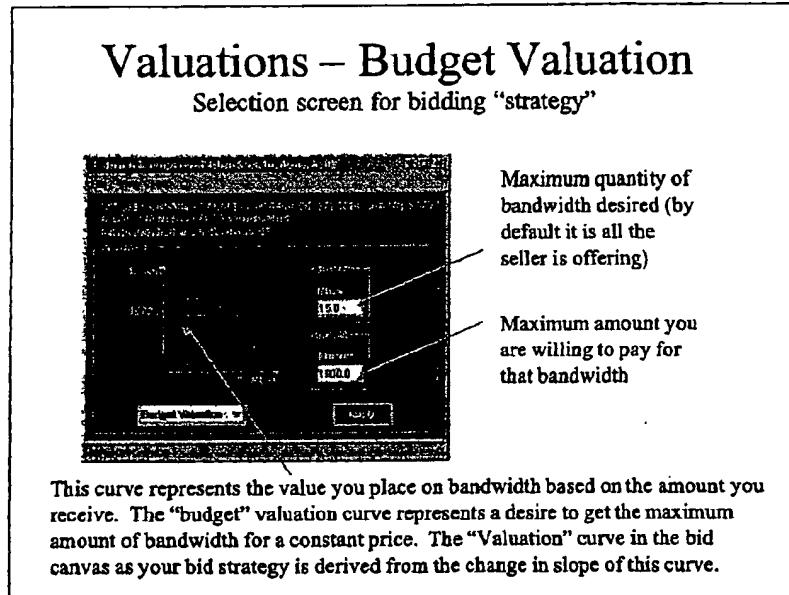


Fig 15(i)

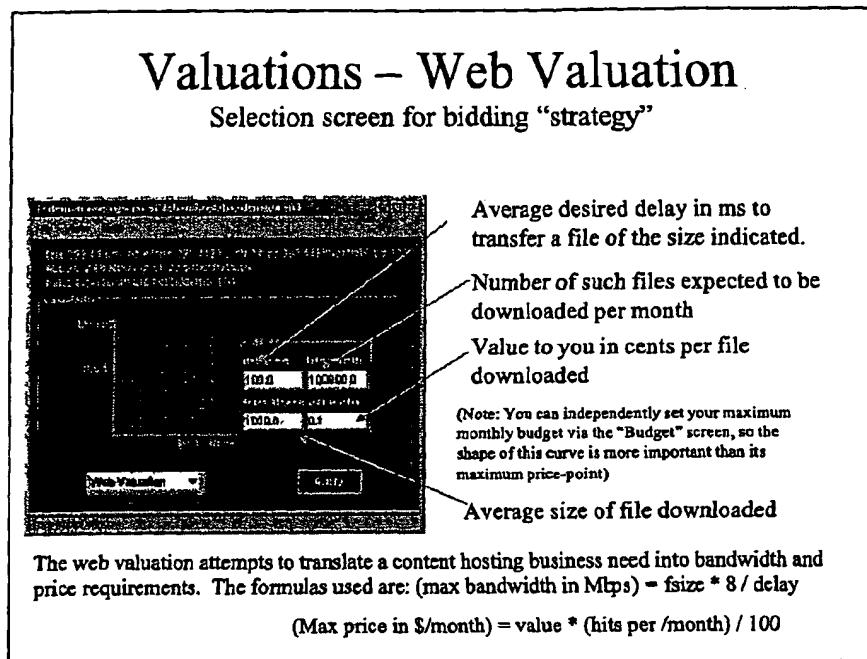


Fig 15(j)

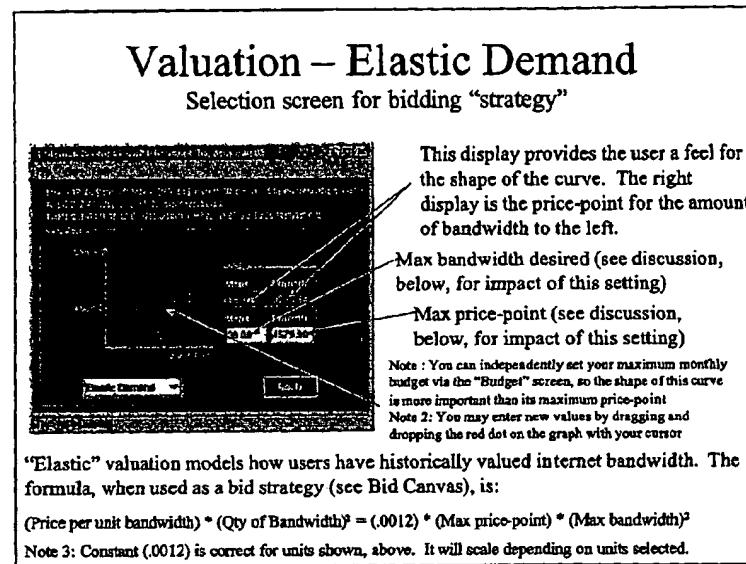
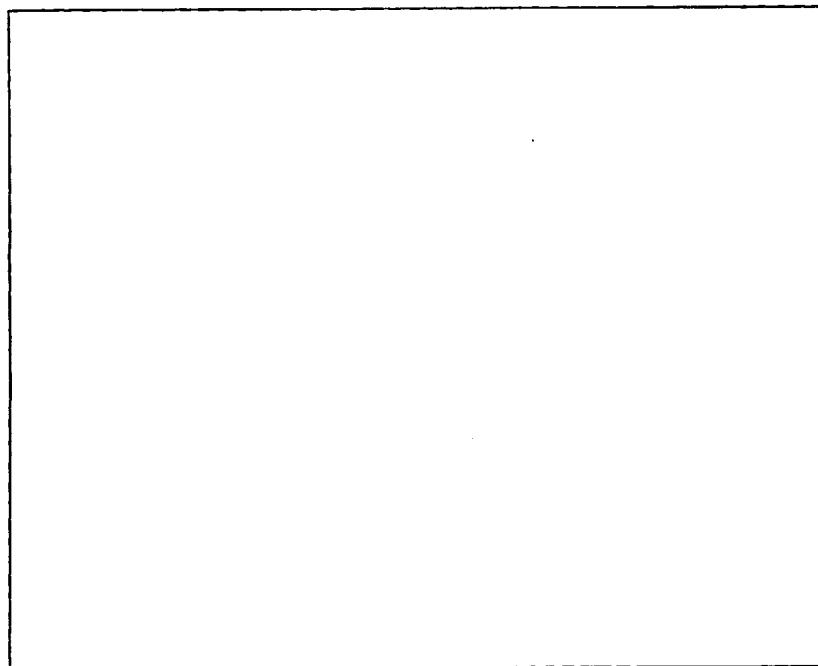
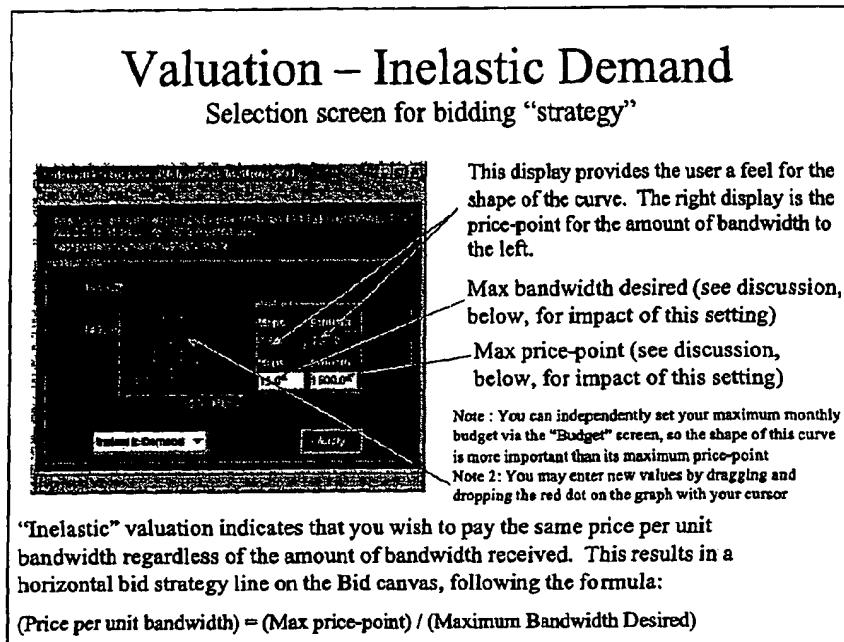


Fig 15(k)





When the elastic bid strategy is combined with the knowledge of the second price auction mechanism, it results in the following behavior:

If the elastic valuation is above the budget line, the agent will do a reverse calculation to determine when it can bid on the valuation line, but obtain the bandwidth on the budget line.

If the elastic valuation is below the budget line, the agent will continue to ask for the maximum amount of bandwidth at the valuation price and not accept a lesser amount of bandwidth.

Fig 15(1)

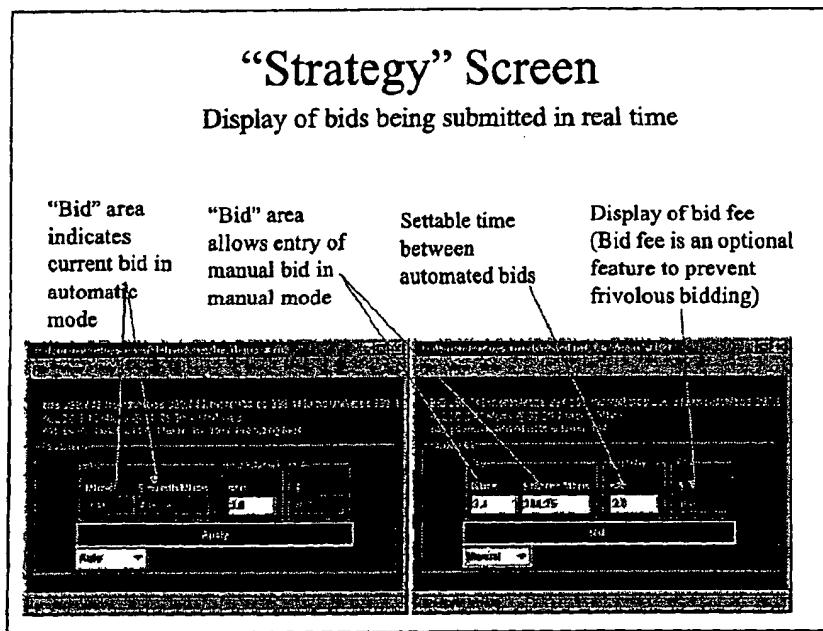


Fig 15(m)

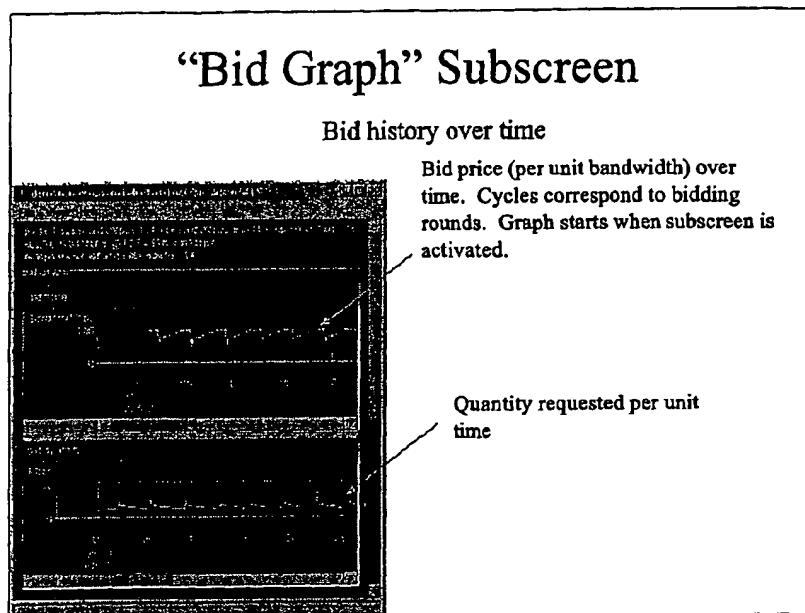


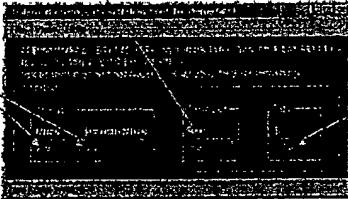
Fig 15(n)

## “Allocation” subscreen

### Results of previous bidding round

If, as is normal, the bidding round is terminated when no bids are received within a configured amount of time, the "Time Left" counter will count down from the configured time, but get reset whenever a bid is received by the Resource Agent, from anyone. When this counter reaches zero, an allocation will be made and a new bidding round will begin after a slight pause to implement the allocation.

**Allocation  
(Quantity and  
Price per unit  
bandwidth)  
received during  
the last bid cycle**



Total amount  
spent during  
this session

Fig 15(o)

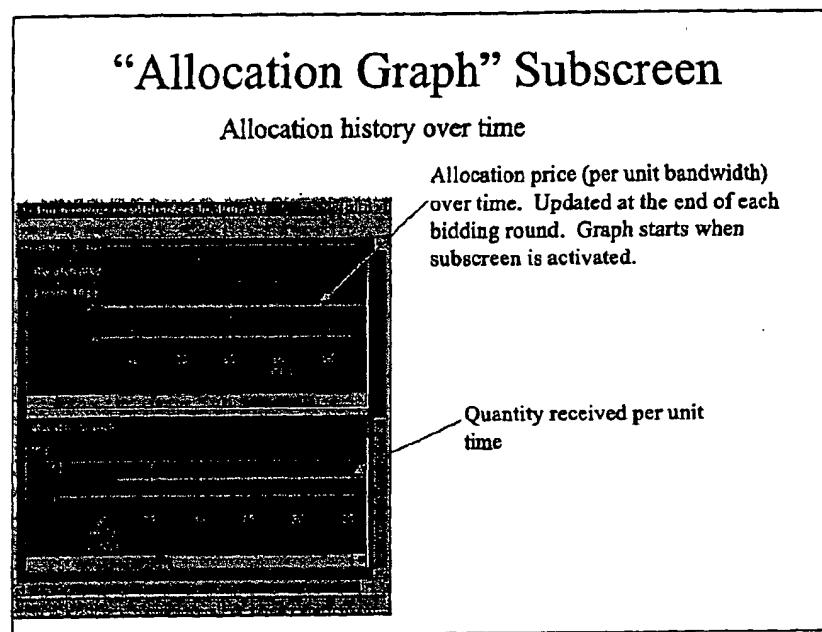


Fig 15(p)

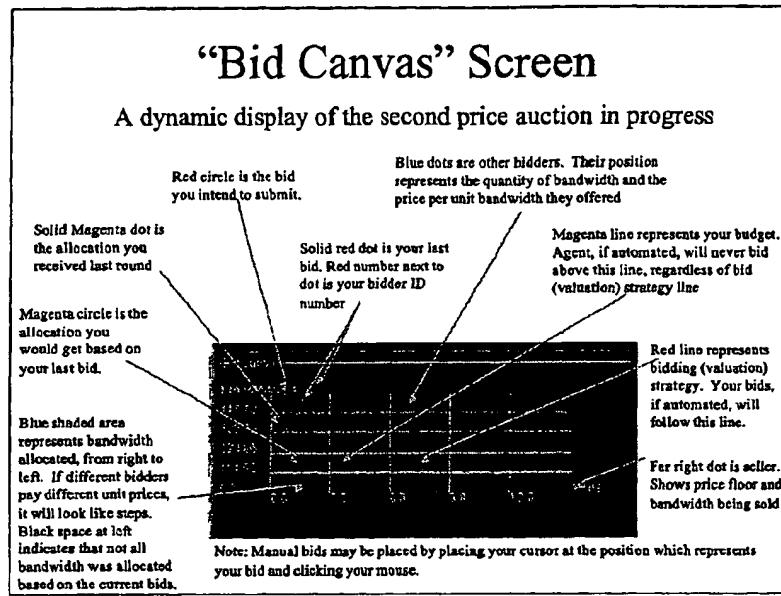
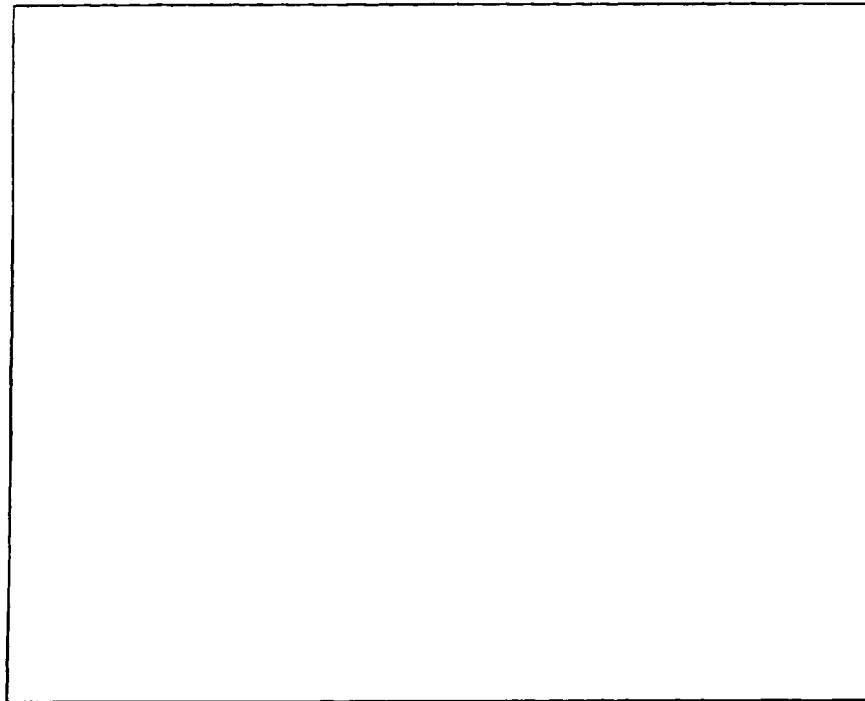
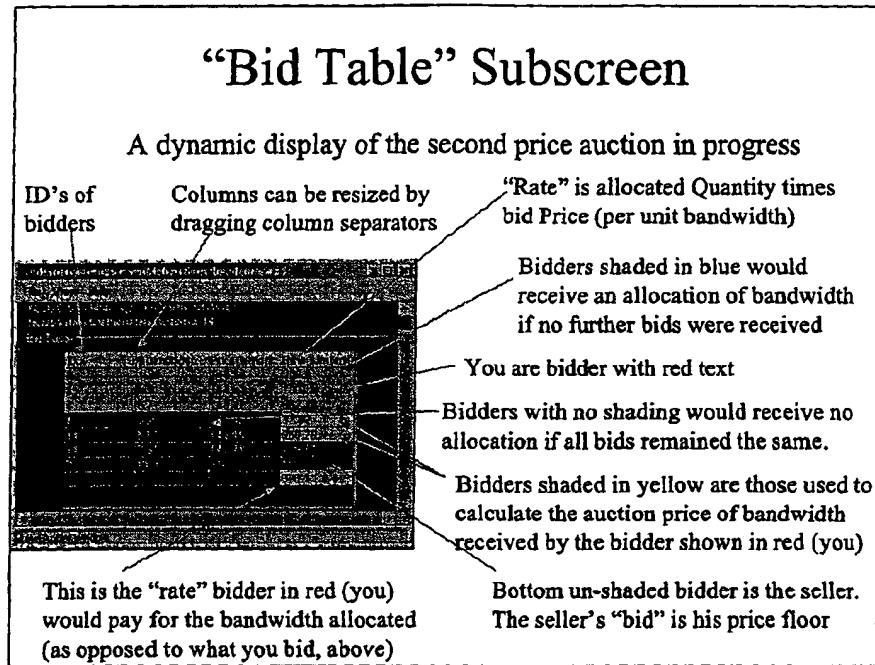


Fig 15(g)





Pig 15(n)

## METHOD AND SYSTEM FOR MARKET BASED RESOURCE ALLOCATION

### RELATED APPLICATIONS

[0001] This application claims priority under 35 U.S.C. §119(e) to U.S. Provisional Application No. 60/203,849, filed May 12, 2000, which is herein incorporated by reference in its entirety.

### BACKGROUND

#### [0002] A. Technical Field

[0003] This application relates to a system and method for real-time resource allocation and, specifically, to a system and method for allowing real-time buyers and sellers to bid for resources and controlling the applicable resources in accordance with the bids.

#### [0004] B. Background of the Invention

[0005] Over the past several years, the Internet has become an important mechanism for conducting business. It has helped reduce business cost and enabled the customized delivery of goods and services. To facilitate exchange of goods and services, electronic commerce technologies have been developed, ranging from simple Internet shopping sites to auction sites such as Yahoo (<http://www.yahoo.com>) and eBay (<http://www.ebay.com>).

[0006] At the same time, a small but growing market for telecommunication network bandwidth has developed. Bandwidth, however, is unlike traditional commodities, goods and services, in that: 1) it is a shared resource that cannot be stored. As such, any bandwidth capacity unused by one party can be made available to all other parties having access to it or it is lost. 2) The resource of bandwidth is consumed in real time. That is, it can occur simultaneously with the buying of the resource. 3) Demand for bandwidth fluctuates very rapidly and tends to be elastic.

[0007] Recently, the need for a dynamic bandwidth commodity market has been recognized. Bandwidth exchanges have been developed between new network companies and Internet service providers. The old system of signing lengthy contracts after weeks or months of negotiation does not move fast enough for buying and selling of many Internet-related resources, such as bandwidth.

[0008] The allocation of information service resources can be viewed as an exchange of commodities. In particular, in the case of bandwidth, Dow Jones has announced plans to launch a bandwidth index that will provide price for long-haul data routes and will enable companies to use these figures to peg the changing process for contracts when they are buying access to networks.

[0009] The emerging telecommunications market call for new mechanisms for real-time trading of network resources, such as bandwidth. In a convention human-based resource trading system, resources for sale are posted, such as on a bulletin board or similar online location. Human beings peruse the posted resources and try to decide which resources their organizations will need in the future. The humans then place bids against each other for the resources. Such human-based resource trading systems usually operate on large amounts of bandwidth at a time and trades are performed periodically, the period being fairly long due to

the limits of human attention span and speed. For example, it would be impractical for a human-base trading system to buy and sell resources in one minute or five minute partitions, since human beings are not capable of such speed. In addition, most human-based resource agents rely on additional human interaction to implement the results of the resource bidding.

### SUMMARY OF EMBODIMENTS OF THE INVENTION

[0010] The present invention provides a platform for resource allocation in real-time. One or more software resource agents interact with software player agents, which are usually both s and seller agents, to reach an agreement on price and quantity allocations for each buyer of that resource (for example, X Mbs of bandwidth for Y units of time, at price Z for buyer A). The s operate in accordance with one or more strategy rules for that agent. A strategy rule tells the what strategy to use in bidding against other agents for particular resources. The s also operate in accordance with valuation rules that tell the how to value a particular resource when bidding (this value is often used as a part of the strategy rule). Seller agents also contain their own strategy and valuation rules, which allow them to decide how much of a resource to offer and how to set a minimum price for the resource. Both player agents (buyer and sellers) and resource agents are aware of a global allocation rule used by the resource agent to allocate a resource between the buyers. In the buyer and seller agents, this allocation rule is often considered in determining strategy.

[0011] In the described embodiment, player agents (buyer and seller) also contain a graphical user interface that allows human beings to set their rules and to control various aspects of the player agent.

[0012] In general, the present invention promotes the sharing of a limited resource, such as bandwidth, buffer space, memory space, storage, or processor time, in a competitive environment. This environment ensures that whomever need the most resource and has the ability to pay will get a share of the resource in accordance with willingness to pay, in addition, the invention adjusts for the changing needs of the participants over time.

[0013] Advantages of the invention will be set forth in part in the description which follows and in part will be apparent from the description or may be learned by practice of the invention. The objects and advantages of the invention will be realized and attained by means of the elements and combinations particularly pointed out in the appended claims and equivalents.

### BRIEF DESCRIPTION OF THE DRAWINGS

[0014] FIG. 1 is a block diagram of an embodiment of the present invention.

[0015] FIG. 2 is a flow chart of a method performed by a resource agent of FIG. 1.

[0016] FIG. 3 is a flow chart of a method performed by a of FIG. 1.

[0017] FIG. 4 is a flow chart of a method performed by a seller agent of FIG. 1.

[0018] FIG. 5 is a chart showing an example of the result of an allocation rule.

[0019] FIGS. 6(a) and 6(b) show an example of a valuation rule.

[0020] FIGS. 7(a) and 7(b) show an example of a strategy rule.

[0021] FIG. 8 shows an example of a flow chart used by an accounting system of an embodiment of the invention.

[0022] FIG. 9(a) is a more detailed block diagram of the embodiment of FIG. 1.

[0023] FIG. 9(b) shows an example of how a bandwidth resource is given to the winning bidder.

[0024] FIG. 10 shows an example of an embodiment of the present invention including a "garage" for player agents.

[0025] FIGS. 11(a)-11(b) are an example of an XML file for a generic player agent.

[0026] FIGS. 12(a)-12(c) shown an example of Java code implementing a strategy rule for a.

[0027] FIGS. 13(a)-13(c) show an example of Java code implementing an allocation rule for a resource agent 104.

[0028] FIG. 14 is an example of an XML file for a generic resource agent.

[0029] FIGS. 15(a)-15(r) show examples of user interfaces for buyer and seller agents.

#### DETAILED DESCRIPTION OF EMBODIMENTS

[0030] Reference will now be made in detail to several embodiments of the present invention, examples of which are illustrated in the accompanying drawings. Wherever practicable, the same reference numbers will be used throughout the drawings to refer to the same or like parts.

[0031] The present invention uses distributed, self-optimizing software agents to perform resource allocation more efficiently than centralized or human-based systems.

[0032] FIG. 1 is a block diagram of an embodiment of the present invention. FIG. 1 includes a software player agent 102, which represents multiple and seller agents. A typical system will, include both buyer and seller agents 102. FIG. 1 also includes a software resource agent 104, a software accounting agent 106, a network control and management agent 108 and a resource 110. It is contemplated that resource 110 can be a number of different resources, including but not limited to: bandwidth, buffer space, memory space, storage, or processor time. As shown in FIG. 1, each player agent (buyer and seller agent) can contain a Graphical User Interface (GUI) 122, one or more valuation rules 124, one or more strategy rules 126, and one or more allocation rules 128. The resource agent 104 also contains the same one or more allocation rules 128.

[0033] s 102 place bids to the resource agent 104, which ultimately decides which of the player agents is awarded a portion of each resource for a predetermined amount of time.

[0034] Resource agent 104 also controls resource 110, based on the results of the bidding, by sending an allocation command to network control and management agent 108. Thus, for example, if the resource is Internet bandwidth, if

buyer A has won X Mbs of bandwidth for 5 minutes, resource agent 104 directs the network control and management agent 108 to give precedence to packets originating at buyer A for the next five minutes or to ensure that the agreed upon bandwidth requirements are met for buyer A's packets for the next five minutes. The mechanism used to communicate the allocation command from resource agent 104 to agent 108 can be any appropriate mechanism. In the example shown, the allocation command will include an identification of the winning buyer or buyers and an identification of the amount of resource allocated and the time period for which it is allocated. Other appropriate formats can be used without departing from the spirit of the invention. In some embodiments, agents 104 and 108 are separate agents (as shown in the figure). In other embodiments, their functions are merged into a single agent. Agents 104 and 108 can be owned and/or controlled by the same entity or by different entities. For example the owner of resource 110 could outsource the market-based allocation to a business partner operating resource agent 104, while keeping the network management and control function in its own hands, by retaining operation of the network control and management agent 108 themselves.

[0035] In the embodiment shown, network control and management agent 108 controls resource 110 to implement the allocation command received from resource agent 104. Thus, agent 108 commits the resource allocated to a player after the resource agent 104 has closed the bidding. Network control and management agent 108 provides a common interface for resource agent 104 for all systems supported. In certain embodiments, there is one agent 108 per resource. Alternately, a single agent can control multiple resources and communicate with multiple resource agents 104. In the embodiment shown, network management and control agent 108 controls resources belonging to other entities (i.e., not to the owner of resource agent 104). In other embodiments, resource 110 might also be under the direct control of the owner of resource agent 104.

[0036] FIG. 1 shows that network control and management agent 108 sends either Simple Network Management Protocol (SNMP) commands or COPS (Common Open Policy Service Protocol) commands to the resource. The SNMP protocol is well known and is described in RFC 1089, RFC 1270, RFC 1303, RFC 1298, RFC 1418, and RFC 1419, which are incorporated herein by reference in their entirety. The COPS protocol is well known and is described in "The COPS (Common Open Policy Service," dated Mar. 5, 1999, available from Adobe Systems, Inc. of San Jose, Calif., and RFC 2748., both of which are incorporated herein by reference in their entirety. Other appropriate resource control protocols can be used without departing from the spirit of the invention.

[0037] Once one or more winning buyers are determined, resource agent 104 alerts accounting agent 106, which keeps track of the winning buyers, as described below in connection with FIG. 9(a). Accounting agent 106 provides a common interface for all accounting systems supported. Agent 106 preferably contains a database handler for each system it supports. Although accounting agent 106 is shown as using one or more of the IHN and SQL database interfaces, any appropriate interface to an accounting database could be used.

[0038] The 102 operates in accordance with one or more strategy rules for that agent. A strategy rule tells the what strategy to use in bidding against other agents for particular resources and, therefore constitutes a bidding mechanism for agents.

[0039] The s also operate in accordance with valuation rules that tell the how to value a particular resource when bidding (this value is often used as a part of the strategy rule). Thus, a valuation rule typically tells an agent how to value each unit of a resource over a range of possible quantities, at a given time. Seller agents also contain their own strategy and valuation rules, which allow them to decide how much of a resource to offer and how to determine a minimum price for the resource. Strategy and valuation can depend on external information, such as accounting information and network congestion.

[0040] The valuation and strategy of player agents (buyer and sellers) and resource agents are aware of a global allocation rule used by the resource agent to allocate a resource between the buyers. In the buyer and seller agents, this allocation rule is often considered in determining valuation and/or strategy. Thus, an allocation rule typically can be thought of as corresponding to a market mechanism. Examples of allocation rules include English auctions (familiar to persons who frequent human-based antique and estate sale auctions), Reverse Price Auctions (such as the main eBay model), Dutch auctions, and continuous bid-ask trading. Examples of allocation rules are found, for example, in 1) J. F. Rosenschein and G. Zlotkin, "Rules of Encounter," MIT Press 1994; 2) PhD thesis of N. Semret, "Market Mechanisms for Network Resource Sharing," Dept. of Electrical Engineering, Columbia University, submitted approximately May 1999; and 3) H. R. Varian, "Economic Mechanism Design for Computerized Agents," USENIX Workshop on Electronic Commerce, July 1995. Each of these three references is incorporated herein in its entirety.

[0041] FIG. 2 is a flow chart of a method performed by resource agent 104 of FIG. 1. In at least certain embodiments, there is more than one resource agent—one for each resource. For example, if an ISP offers several different bandwidths, each bandwidth might be considered a separate resource, controlled by a separate resource agent 104. Resource agents preferably run within servers that can be distributed over a network. Alternately, one or more resource managers 104 can reside on the same system.

[0042] As shown in FIG. 2, resource agent 104 receives 202 one or more bids for resource 110 from one or more s 102. Such bids will usually include at least quantity and price values. If a buyer is outbid, the resource agent notifies 204 the buyer, unless the trading period is over 206. Once the trading period is over, the resource agent notifies 208 the winning or agents and proceeds to send an allocation command 210 to network control and management agent 108 that will cause agent 108 to control the resource in accordance with the winning bids.

[0043] FIG. 3 is a flow chart of a method performed by player/buyer 102 agent of FIG. 1. First, the is apprised of potential resources available for bidding. This is accomplished either by the player requesting current resource auction information from a centralized directory service (not shown) or by the player registering with the resource agent and periodically being sent information about current bid-

ding. With a directory service, a player agent 102 queries the directory service to find the location (e.g., in the form of a URL) of resource agents 104, garages (see FIG. 10), and other player agents 102. The decides 302 whether to bid on a particular unit or resource 110 and sends 304 a bid to the resource agent. If the receives a notice that he has been outbid, control returns to element 302 and the agent decides whether to continue bidding.

[0044] In element 302, the uses its knowledge of the system market allocation rule and of its own valuation rules to determine how much it is willing to pay for resources at a given time and uses its own strategy rules to determine whether to bid on available resources. If the includes a GUI, a human being can visualize the market using various known graphs, charts or similar graphics. A user can also use the GUI to change the strategy and valuation rules used by the agent.

[0045] It should be noted that certain embodiments include a special type of player, called a broker agent, which buys resources with the intent of reselling those resources. Thus, for example, a broker agent has no user for the resource (such as bandwidth) itself, since the broker is not an ISP or similar entity in need of bandwidth. Instead, the broker arranges for third party customers, such as ISPs, to receive the benefit of the resources the broker has bid for and won, by reselling them through another resource agent instance, and the broker keeps the difference between the buying and selling price. In such a case, the new resource agent 104 informs the network control and management agent 108 which party is to benefit from the resource bid for by the broker. FIG. 4 is a flow chart of a method performed by a player/seller 102 agent of FIG. 1. A seller agent first notifies 402 the resource agent that it has one or more units of a resource to sell (for example, 1 Mbs for 5 minutes or 10 minutes of processor time). Once the bidding is over, seller agent 102 will receive from resource agent 104 a notification 404 of the winning bidder or bidders. In at least one embodiment, the seller agents collect the payments from the buyers based on accounting information retrieved from the accounting agent. 106.

[0046] FIG. 5 is a chart showing an example of the result of a Progressive Second Price Auction (PSP) allocation rule. Other allocation rule types, as discussed above, can also be used, as specified by the particular allocation market strategy implemented in a particular system. FIG. 5 shows how resources are allocated and prices set once the bidding is over and it is determined that there are not enough resources to satisfy all the bidders.

[0047] In the PSP auction of FIG. 5, bidder A bids \$3 for 50 resource units; bidder B bids \$2 for 30 resource units; bidder C bids \$1 for 30 resource units; and bidder D bids \$0.50 for 20 resource units. At the close of bidding, resource agent 104 allocates the resources as follows: The 100 available resource units are apportioned between the bidders until the resource is gone. Thus, the high bidder A is allocated all of the resource that he wants, as is the second high bidder B. Bidder C only gets 20 or the 30 resource units that he wanted and bidder D gets nothing, since there are no more resource units to allocate after partially fulfilling bidder C's wants.

[0048] The resource agent 104 determines the amount that the bidders in the PSP auction are charged as follows: For

each bidder, the resource agent determines the value of the bidder's resource if that bidder had not participated, and charges the bidder a price based on this determination.

[0049] Thus, if bidder A had not participated, his 50 units would have been allocated as follows:

[0050] 10 units to bidder C (to make up C's shortfall)

[0051] 20 units to bidder D (to give D his requested number of units)

[0052] The remaining 20 of bidder A's units would not have been allocated.

[0053] The cost to bidder A is determined as follows:

[0054] The cost of a resource unit to bidder C is \$1/30.

[0055] Thus, the value that would have been given to bidder C:  $10 \times (\$1/30) = \$0.33$

[0056] The cost of a resource unit to bidder D is \$0.50/20.

[0057] Thus, the value that would have been given to bidder D:  $20 \times (\$0.50/20) = \$0.50$

[0058] Thus, the cost of A's resources had A not participated is  $\$0.33 + \$0.50 = \$0.83$ . Bidder A is charged this amount for his 50 units of resource.

[0059] Similarly, if bidder B were not present, his 30 units would have been allocated as follows:

[0060] 10 units to bidder C (to make up C's shortfall)

[0061] 20 units to bidder D (to give D his requested number of units)

[0062] The cost to bidder B is determined as follows:

[0063] The cost of a resource unit to bidder C is \$1/30.

[0064] Thus, the value that would have been given to bidder C:  $10 \times (\$1/30) = \$0.33$

[0065] The cost of a resource unit to bidder D is \$0.50/20.

[0066] Thus, the value that would have been given to bidder D:  $20 \times (\$0.50/20) = \$0.50$

[0067] Thus, the cost of B's resources had B not participated is  $\$0.33 + \$0.50 = \$0.83$ . Bidder B is charged this amount for his 30 units of resource.

[0068] Similarly, if bidder C were not present, his 20 units would have been allocated as follows:

[0069] 20 units to bidder D (to give D his requested number of units)

[0070] The cost to bidder C is determined as follows:

[0071] Value that would have been given to bidder D:  $20 \times (\$0.50/20) = \$0.50$

[0072] Thus, the cost of C's resources had C not participated is \$0.50. Bidder C is charged this amount for his 20 units of resource.

[0073] It should be noted that FIG. 5 shows only how resources are allocated after bidding is closed. An allocation rule also includes within it rules or explanations of how the

auction itself should be conducted. For example, a PSP auction generally lasts for a predetermined amount of time (for example, five minutes). While the bidding is open, resource agent 104 collects all bids received from the s 102 and saves them in a bidlist data structure (e.g., a linked list). The bidlist data structure indicates which bid is the most recent bid for each 102.

[0074] As bids are received from the s 102 by the resource agent 104, the resource agent 104 transmits the bids received to the other agents 102, so that all agents know what all other participating agents are bidding. Because each 102 has knowledge of the allocation method, each 102 can apply its strategy and valuation rules to determine whether that agent is going to be allocated the resources on which it has bid. The agent, applying the allocation, strategy, and valuation rules, determines whether it should bid again. In a PSP auction, bidding usually stabilizes after a few minutes. In some embodiments, resource agent 104 does not hold the auction open for a predetermined time, but instead waits a predetermined amount of time after the bidding has stabilized to make sure that no other bids are received. In some cases, resource agent 104 announces to the s 102 that bidding will close in a certain number of minutes or seconds. In some cases, if a bid is received during this time period, the auction is kept open a bit longer.

[0075] Thus, an allocation rule (known to all agents) also includes information about how the auction will be conducted by resource agent 104, including, for example: how long an auction will last, if the auction has no set time period, what are the conditions for the auction to close. In addition, as described above, the allocation rule includes rules describing how price and quantities are assigned after the auction has closed.

[0076] In a preferred embodiment of the invention, auctions last 5 minutes, although other periods of time could be used and these periods of time could be either variable or user-settable. In a preferred embodiment, the bandwidth resource being auctioned during a current auction is allocated immediately and another auction is begun immediately. Thus, an auction occurs roughly every five minutes for the bandwidth that will be used by the buyers during the next five minutes. Other embodiments may not auction all bandwidth for immediate use.

[0077] The PSP auction model is described further in A. A. Lazar and N. Semret, "Design and Analysis of the Progressive Second Price Auction for Network Bandwidth Sharing," Telecommunications Systems, Special issue on Network Economics, which is attached hereto as Appendix A and forms a part of this application.

[0078] Other examples of allocation rules include, but are not limited to Hold Option Auctions, which are discussed, for example, in the PhD thesis of N. Semret, "Market Mechanisms for Network Resource Sharing," Dept. of Electrical Engineering, Columbia University, submitted approximately May 1999, Chapter 4 of which (28 pages) is attached hereto as Appendix B and which forms a part of this specification. The Hold Option is a concept for advance price and quantity guaranteed reservations of network resources in a real-time market environment. Periodic auctions (progressive second price, or other) among arrivals grouped in batches give rise to the spot market of capacity changes. A reservation guaranteeing access for an arbitrary

duration with a capacity piece below the bid can be made at any time before or during service. This eliminates the risk (which is inherent on the spot market) of losing resources to higher bidders before service completion. The reservation is defined as a hold option, and is analogous to derivative financial instruments such as options and futures integrated over time. Based on a heavy traffic diffusion model, reservation fees can be computed as the fair market price of a hold option. In at least one embodiment, special player agents 102 are allowed to place such hold options, thus providing a guaranteed reservation of a network resource for an arbitrary duration.

[0079] FIGS. 6(a) and 6(b) show examples of valuation rules. In FIG. 6(a), a valuation rule is formed of pairs of quantity/price values. In FIG. 6(b), a valuation rule is formed as a function of various input variables. These input variables can include any or (but are not limited to): the number of hits on a web site the average file size downloaded (and the bandwidth needed to service those files); the expected delay or expected latency, and the value of each hit. Valuation can also be time dependent (e.g., higher valuations are assigned during peak usage times when more bandwidth is needed) and the network state (e.g., more bandwidth is needed if the network is congested).

[0080] It should be noted that there are engineering tradeoffs for the type of valuation rule used. Because a low-level valuation rule requires more inputs, which require more time and effort to collect and receive, a low-level description may require a large amount of data to be transferred in order for the agent to be able to bid in accordance with the low-level valuation rule. On the other hand, a simple, high-level valuation rule reduces the ability of an agent to make an optimum bid because the agent is operating with less information. Either implementation can be correct for a given circumstance, depending on the needs of the particular player agent and the limitations of its system and network.

[0081] FIGS. 7(a) and 7(b) shows examples of strategy rules. FIG. 7(a) shows a simple rule set, where the first precedent is to identify the type of allocation system being used. Once the allocation system is identified, the 102 applies a set of predefined conventional rules to determine whether it should bid (or bid again). FIG. 7(b) shows an example where the strategy is based on a user-defined function. In the example, if the function reaches a threshold value, the agent 102 will bid (or bid again). Other examples of strategy rules decide not just whether the agent should bid, but how much the agent should bid, in accordance with the allocation rule being used by the system and in accordance with factors specific to that agent (such as, for example, those factors discussed above in relation to valuation rules).

[0082] Certain strategy rules are very simple and involve bidding constant amount. Such simple strategy rules may result in uneven amounts of bandwidth being won. Another example strategy rule is periodic bidding, in which a buyer agent enters auctions periodically.

[0083] FIG. 8 shows an example of a flow chart used by an accounting system of an embodiment of the invention. The accounting system is notified whenever a resource agent closing bidding on a resource unit and keeps track 802 of the winning bids and resulting allocations. Periodically, the

accounting system bills 804 the seller a percentage of the resources sold, which is received by the owner of the resource agent for operating the resource agent and account agents of the Merkato platform.

[0084] FIG. 9(a) is a more detailed block diagram of the embodiment of FIG. 1. It will be understood that FIG. 9(a) details only one possible way to implement the present invention and that the description herein is not to be taken in a limiting way with regard to operating systems, protocols, programming languages, etc. The example shown is implemented in Java using the World Wide Web, but the invention is not limited to such a system. In fact, the invention can be implemented on any appropriate computers and networks, using any appropriate programming language, hardware, software, and operating system. Parts of the system can be implemented in software, firmware, or hardware, as needed.

[0085] FIG. 9(a) includes four layers: an operating system (OS) layer 902, a Java layer 904, a Merkato layer 906, and a diffex layer 108. In the described embodiment, the OS layer 902 and the Java layer 904 are conventional and will not be described herein. Other embodiments may make changes to one or more of layers 902 and 904 to enhance the performance of the system, for example. In this example, the use of Java layer 904 makes the system platform independent. Thus, the Merkato layer 906 can run on any computer or computing device (such as a wireless device, pager, cell phone, or Internet appliance)

[0086] Layer 906 allows a player agent 102 to be executed on the client side, from a Java application or an applet executing in a Web browser. Alternately (or in addition), a player agent 102 can be executed as a servlet on a web server, which is the "garage" environment of FIG. 10.

[0087] Layer 908 enables real-time resource markets between peering ISPs. The layer 908 allows ISPs to buy and sell resources, such as bandwidth, from each other in real-time. Layer 908 auctions in real-time the outgoing bandwidth on each ISP's line out of the exchange, ensuring that at all times, the bandwidth goes to the buyer with the highest value for it. Layer 908 also allows for buying and selling in advance (i.e., making reservations with guaranteed capacity and capped prices) through a derivative market of options, in effect enabling the trading of risk and hedging, for original ISP buyers and sellers, as well as for purely financial players.

[0088] FIG. 9(b) shows an example of how a winning buyer ISP 952 is given a resource, such as bandwidth. In FIG. 9(b), a seller ISP 956 has a seller 902 agent running on a diffex client. As discussed above, the seller agent determines that the ISP has bandwidth to sell (e.g., through user input via the seller agent GUI) and makes that bandwidth available for auction by sending a message to resource agent 904 (see numeral 1 in a circle). The buyer ISPs 952, 954 have a 902 running on a diffex client. The s bid on the bandwidth in accordance with their allocation rules, strategy rules, and valuation rules, as discussed above (see numeral 2 in a circle). For certain types of allocation models, the agents also receive information about bids during the auction (not shown). Once the auction is concluded, resource agent 904 sends an allocation command to network control and management agent 908 (numeral 3 in a circle), which in

turn controls a router 970 of the seller ISP so that the winning buyer ISP receives its bandwidth (see numeral 4 in a circle).

[0089] Resource agent 104 in layer 908 interfaces with the resource (e.g., with the router 970 of the seller ISP) to allocate the bandwidth to the winning bidder. Specifically, resource agent 904 interfaces with control agent 908 to send commands to the router of the seller ISPs. If, for example, the resource is bandwidth, allocation to the winning bidder can be effected by one of the following or by any other appropriate method:

[0090] a) the router 970 is given a set of class-based weighted-fair queuing parameters to be used by the router to control packet queuing in the router so that the winning ISP buyers are assured of receiving the bandwidth which they was allocated by the resource agent. These queuing parameters will give priority to the winning bidders when packets from the winning ISPs are sent to the seller ISP's router.

[0091] b) the router is given a set of committed access-rate parameters to be used by the router to limit and shape traffic assure each buyer the bandwidth which it is allocated, or

[0092] c) capacity within an MPLS (Multiprotocol Label Switching) tunnel between two points in the seller's network.

[0093] Thus, as shown in FIG. 9(b), packets 960 from the winning buyer ISP(s) are routed through the seller's router and on to the seller ISP's network 956, from which they are delivered to their destination. In at least one embodiment, the winning ISPs are aware that they have won and use existing routing protocols to ensure that they direct packet traffic to the seller ISP's router. In other embodiments, the resource agent informs routers in the winning ISPs of the needed routing change using known routing protocols.

[0094] In the described embodiment, the resource agent is always executed on a Web server. Each resource agent 104 runs as a servlet on a Web server. This architecture is scalable because resource agents 104 can be distributed to run on any Web servers supporting the concept of a servlet.

[0095] In the described embodiment, communications between a player agent (either a buyer or a seller) 102 and a resource agent 104 are performed via a resource agent proxy using any of http extensions, native TCP protocol, or Java's remote method invocation (rmi). All communications are secured through an appropriate mechanism such as the secured socket layer (SSL). As shown in FIG. 9(a), each agent (player and resource) has an allocation rule object 128. The player agents 102 also have valuation rule objects 124, strategy rule objects, 126, and a GUI object 122. In addition, the resource agents 104 have networking and accounting drivers for interfacing with external support systems. Further security is provided through the implementation of specific allocation rules that protect the stability of the system. Specifically, each user is required to pay a bid fee to resource agent 104 for every bid sent. The implementation of a bid fee is intended to prevent users from trying to

artificially destabilize the resource price and to prevent a "man in the middle" attack, which is defined as a third party intercepting a bid from another agent and keeping sending the same bid over and over. In addition, timestamps are preferably used to discriminate every bid. Thus, if a bid has a previously used timestamp, resource agent 104 will ignore that bid.

[0096] In the described embodiment, communication between agents is effected using http, thus bypassing many problems associated with firewalls, proxies, and other middleware.

[0097] FIG. 10 shows an example of an embodiment of the present invention including a "garage" for player agents 102. The garage 130 is a component on a web server that serves the purpose of storing agents and enabling their execution remotely from a user's computer. The garage contains an "attendant" program 131 whose job is to help mobile agents find a parking place in the garage and to ensure proper agent execution when the agents want to run autonomously. Garage 130 performs the function of a distributed database to store the agents 102 and provides a distributed processor (not shown) to execute the agents. Garage 130 can be located on the same system as resource agent 104, but can also be located on a different machine.

[0098] The attendant is an example of a multi-agent. Multi agents coordinate multiple simple agents to act together. Multi-agents can be used to form coalitions of agents, using the attendant to bid for aggregated resources on behalf of the coalition. In the described embodiment, the agents communicate with the attendant to let the attendant know that they need resources. The multi-agent aggregates the various types of resources requested (e.g., different connection speeds or different bandwidths) and uses the allocation rule and its own strategy and valuation rules to bid on behalf of the agents. If the multi-agent wins, the resource is divided between the agents and the attendant so informs the resource agent 104, which allocates the bandwidth accordingly. A broker is an example of a multiagent, coordinating buyer and seller agents for a common objective to act as a profit-maximizing reseller.

[0099] In the described embodiment, agent mobility to the garage is provided through XML. The syntax and semantics of an agent is described using XML. For example, the semantics of an agent can include its allocation, strategy and valuation rules. Each agent can be transferred to the "garage" 130 by generating its XML description. Once an agent is provided in this form, it can be re-instantiated either in a garage 130 or in a user's computer in an applet or a java application.

[0100] FIGS. 11(a)-11(b) are an example of an XML file for a generic player agent 102. This XML would be used, for example, to send the agent 102 to a client for execution in garage 930. Each of the XML tags (indicated by < > brackets) identifies an attribute of the agent that is to be activated in the garage. It will be understood that the XML of FIG. 11 is shown for purposes of example only and that other embodiments of the invention may use more or fewer tags and/or different tags than those shown in FIG. 11.

[0101] FIGS. 12(a)-12(c) shows an example of Java code implementing a strategy rule for a 102. This example implements a strategy in accordance with the PSP auction model described above. As shown in section 1234 of FIG. 12(c), an agent using this strategy will submit a new bid only if the new bid determined in the rule is increased by at least a calculated value epsilon. Note that this strategy rule calls a valuation rule in line 1232 of FIG. 12(c). This valuation rule implements a PSP valuation method.

[0102] FIGS. 13(a)-13(c) show an example of Java code implementing an allocation rule for a resource agent 104. This example looks at a number of received bids in a bidlist data structure and computes an allocation (similar to that of FIG. 5) given the current bids on the bid list in accordance with a PSP auction allocation model.

[0103] FIG. 14 is an example of an XML file for a generic resource agent 104. This XML would be used, for example, to send the resource agent 104 to a web server. Each of the XML tags (indicated by < > brackets) identifies an attribute of the agent that is to be activated on the server side.

[0104] FIGS. 15(a)-15(r) show examples of user interfaces for buyer and seller agents. FIGS. 15(a) and 15(b) show an example of an html GUIs that provides static information to a user. FIGS. 15(c) and 15(d) show an example of GUIs implemented as a Java applet. The information provided by these interfaces is continuously updated in real-time or periodically. FIGS. 15(e)-15(r) show an example of an advanced GUI, which is preferably also implemented as an agent. Particular buyer and seller agents may have one, none, or all of the particular GUIs shown here, or may have GUIs providing other relevant information not shown here. Note that several of these GUIs allow a human user to choose the valuation and/or strategy rules and to determine when and how to bid. It should be understood that in the preferred embodiment, buyer agents are capable of bidding on their own. Other embodiments may contain agents that bid only at the direction of human beings.

[0105] Accordingly, the present invention is intended to embrace all such alternatives, modifications and variations as fall within the spirit and scope of the appended claims and equivalents.

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**APPENDIX A**

**Docket No.: 61624-04980**



# Design and Analysis of the Progressive Second Price Auction for Network Bandwidth Sharing\*

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## Abstract

We present the Progressive Second Price auction, a new decentralized mechanism for allocation of variable-size shares of a resource among multiple users. Unlike most mechanisms in the economics literature, PSP is designed with a very small message space, making it suitable for real-time market pricing of communication bandwidth. Under elastic demand, the PSP auction is incentive compatible and stable, in that it has a "truthful"  $\epsilon$ -Nash equilibrium where all players bid at prices equal to their marginal valuation of the resource. PSP is economically efficient in that the equilibrium allocation maximizes total user value. With simulations using a prototype implementation of the auction game on the Internet, we investigate how convergence times scale with the number of bidders, as well as the trade-off between engineering and economic efficiency. We also provide a rate-distortion

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theoretic basis for valuation of bandwidth, which leads naturally to the elastic demand model that is assumed in the analysis of the mechanism.

**Keywords:** resource allocation, auctions, game theory, mechanism design, network pricing.

## 1 Introduction

Communication networks are characterized by what economists call externalities. The value a user gets from the network depends on the other users. The positive externalities are that a communication network is more valuable if more people are connected. The negative externalities are that resources are shared by users who – because of distance, population size, or selfishness – cannot or will not coordinate their actions sufficiently to achieve the most desirable allocation of resources. The recognition of this reality in many aspects of networks and distributed computations has lead in recent years to the emergence of game theoretic approaches in their analysis and design [23, 9, 25, 33, 15, 16].

Prices, whether they relate to “real money” in a public network or “funny money” (based on quotas) in a private system, play a key role as allocation control signals. In the former case, this role is of course intimately tied to another, which is to allow a network provider to remain in business [7].

The telephone system and the current Internet represent two extremes of the relationship between resource allocation and pricing. The resources allocated to a telephone call are fixed, and usage prices are based on the predictability of the total demand at any given time. On the Internet, the current practice of pricing by the maximum capacity of the user’s connection (flat-rate pricing) decouples the allocation (actual use) of resources from the prices.

In the emerging multiservice networks (ATM, Next-Generation Internet), neither of these approaches are viable. The former because of the wide and rapidly evolving range of applications (including some which adapt to resource availability) will make demand more difficult to predict. And the latter because, once the flat fee is paid, there are no incentives to limit usage since increasing consumption benefits the user individually, whereas limiting it to sustainable levels brings benefits which are shared by all. This makes it vulnerable to the well-known “tragedy of the commons”. With flat pricing alone, the tendency is toward increasing congestion which chases away high-value users, or increasing prices which exclude low-value users [6], in both cases leading to decreased network revenue.

Thus there is a need to develop new approaches to pricing of network resources. Among the requirements are: sensitivity to the range of resource requirements (either through a sufficiently rich range of traffic classes which are priced differently, or by allowing users to explicitly quantify resource requirements); prices must be dynamically responsive to unpredictable demand (market based system); perhaps most importantly the pricing architecture should constrain as little as possible the efficiency trade-offs of the policies.

Indeed, the fundamental issue in designing pricing policies is the trade-off between engineering efficiency and economic efficiency. This trade-off, which is more or less constrained by the underlying network technology, has many dimensions, including:

- how much measurement (from usage to capacity pricing),
- the granularity of differently priced service offerings (e.g. number of traffic classes),
- the level of resource aggregation – both in time and in space – at which pricing is done (per packet/cell or per connection, at the edge of the network or at each hop), and
- the information requirement (how much *a priori* knowledge of user behavior and preferences is required/assumed by the network in computing prices).

An approach which achieves economic efficiency is the smart-market approach of [19], wherein each packet contains a bid, and if it is served, pays a clearing price given by the highest bid among packets which are denied service (dropped). This approach is incentive compatible in that the optimal strategy for a (selfish) user is to set the bid price in each packet equal to the true valuation. Each node in the network becomes an efficient market, but the engineering cost (sorting packets by bid price, as well as per-packet and per-hop accounting) could be significant if line speeds are high relative to the processing power in the router. In [14], users are charged according to a combination of declared and measured characteristics of traffic. By taking an equivalent bandwidth model of resource utilization, and assuming appropriate traffic models, a menu of pricing plans indexed by the declared traffic can be offered which encourages users to make truthful declarations (e.g. of the mean rate), and also encourages the users' characterization efforts to be directed where they are most relevant to the network resource

allocation. As the pricing is relative, [14] does not aim to address the problem of determining the actual monetary values of the market price (that users would be willing to pay). Another pricing scheme which incorporates multiplexing gain is formulated in [12]. These and a number of other schemes are summarized in [11], in a comprehensive view of the connection establishment process, which identifies the user-network negotiation as the key "missing link" in network engineering/economic research. In terms of our taxonomy of the previous paragraph, this is part of the information requirement trade-off. Indeed, in the absence of formal mechanisms to deal with the information problem, complex and (at least intuitively) undesirable things happen. For example, some providers offer expensive "front of the book" rates to uninformed customers, and lower "back of the book" rates to informed customers who may be about to defect to another carrier (see [7] and also the recent wars between AT&T and MCI in consumer long-distance service in the United States). In [34], it is argued that architectural considerations such as where charges are assessed should take precedence over the pursuit of optimal efficiency, and edge pricing (spatial aggregation in terms of our taxonomy) is proposed as a useful paradigm.

In this paper, we propose a new auction mechanism which accommodates various dimensions of the engineering-economics trade-off. The mechanism applies to a generic arbitrarily divisible and additive resource model (which may be equivalent bandwidth, peak rate, contract regions, etc., at any level of aggregation.) It does not assume any specific mapping of resource allocation to quality of service. Rather, users are defined as having an explicit monetary valuation of quantities of resource, which the network doesn't or can't know *a priori*. Thus, in terms of our trade-off taxonomy, this mechanism aims for unlimited granularity, flexibility in the level of aggregation and minimal information requirement.

In the most likely auction scenario, users would be aggregates of many flows data flows for which bulk capacity is being purchased for e.g. Virtual Paths, Virtual Private Networks, or edge capacity [2, 31].

We begin in Section 2 by formally presenting the design of our Progressive Second Price auction mechanism for sharing a single arbitrarily divisible resource, and relating it to classical mechanism design from the economics literature. In Section 3, after describing our model of user preferences and the elastic demand assumption, we prove that PSP has the desired properties of incentive compatibility, stability, and efficiency. The section concludes with simulation results on the convergence properties, and the efficiency trade-offs. Appendix A describes an information theoretic basis

for valuations of the type that are assumed in the analysis of Sections 3, as one possible justification.

## 2 Design of an Auction for a Divisible Resource

### 2.1 Message Process

Following [36], it is useful to expose the design in terms of its two aspects: realization, where a message process that enables a certain allocation objective is defined; and Nash implementation, where allocation rules are designed with incentives which drive the players to an equilibrium where the (designer's) desired allocation is achieved.

In this section we define the message process. Here we make the fundamental choice which will constrain the subsequent aspects of the design. Our first concern here is with engineering. For the sake of scalability in a network setting, we shall aim for a process where a) the exchanged messages are as small as possible, while still conveying enough information to allow resource allocation and pricing to be performed without any a-priori knowledge of demand (market research, etc.); and b) the amount of computation at the center is minimized.

Given a quantity  $Q$  of a resource, and a set of players  $\mathcal{I} = \{1, \dots, I\}$ , an auction is a mechanism consisting of: 1) players submitting bids, i.e. declaring their desired share of the total resource and a price they are willing to pay for it, and 2) the auctioneer allocating shares of the resource to the players based on their bids.

Player  $i$ 's bid is  $s_i = (q_i, p_i) \in \mathcal{S}_i = [0, Q] \times [0, \infty)$ , meaning he would like a quantity  $q_i$  at a unit price  $p_i$ . A bid profile is  $s = (s_1, \dots, s_I)$ . Following standard game theoretic notation, let  $s_{-i} \equiv (s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_I)$ , i.e. the bid profile of player  $i$ 's opponents, obtained from  $s$  by deleting  $s_i$ . When we wish to emphasize a dependence on a particular player's bid  $s_i$ , we will write the profile  $s$  as  $(s_i; s_{-i})$ .

The allocation is done by an allocation rule  $A$ ,

$$\begin{aligned} A : \quad \mathcal{S} &\longrightarrow \mathcal{S} \\ s = (q, p) &\longmapsto A(s) = (a(s), c(s)), \end{aligned}$$

where  $\mathcal{S} = \prod_{i \in \mathcal{I}} \mathcal{S}_i$ .

The  $i$ -th row of  $A(s)$ ,  $A_i(s) = (a_i(s), c_i(s))$ , is the allocation to player  $i$ : she gets a quantity  $a_i(s)$  for which she is charged  $c_i(s)$ . Note that  $p$  is a price per unit and  $c$  is a total cost.

An allocation rule  $A$  is feasible if  $\forall s$ ,

$$\sum_{i \in I} a_i(s) \leq Q$$

and  $\forall i \in I$ ,

$$\begin{aligned} a_i(s) &\leq q_i, \\ c_i(s) &\leq p_i q_i. \end{aligned}$$

**Remark a:** The above formulation is a generalization of what is usually meant by an auction. The latter is the special case where  $a_w(s) = Q$  for some winner  $w \in I$  and  $a_i(s) = 0, \forall i \neq w$ , i.e. the sale of a single indivisible object to one buyer, for which the theory is well developed [22, 24]. In our approach, allocations are for arbitrary shares of the total available quantity of resource. Equivalently, one could slice the resource into many small units, each of which is auctioned as an indivisible object. But in a practical implementation of auctions for sharing a resource, a process of bidding for each individual unit would result in a tremendous signaling overhead. More importantly, since the users would be bidding on a discrete grid of quantities, analytical predictions of outcomes could be misleading since they could be sensitive to the particular choice of grid<sup>1</sup>.

**Remark b:** Most of the mechanism design literature in Economics makes use of the following “Revelation Principle”:

Given any feasible auction mechanism, there exists an equivalent<sup>2</sup> feasible direct revelation mechanism which gives to the seller and all bidders the same expected utilities as the given mechanism.  
([24], Lemma 1)

In this sharing context, a direct revelation mechanism would be one where each user message consists of the user's type, which is the valuation<sup>3</sup> of the resource over the whole range of their possible demands, i.e. a function  $\theta_i : [0, Q] \rightarrow [0, \infty)$ , and the budget (see Section 3.1). A consequence of revelation principle is that the mechanism designer can restrict her attention to direct revelation mechanisms, find the best mechanism in terms of

<sup>1</sup>For a more detailed discussion of this point, see [4] p. 34, and references therein.

<sup>2</sup>By equivalent, in [24] it is meant that, at some equilibrium, all players get the same utility. There may be other, possibly ill-behaved, equilibria.

<sup>3</sup>The valuation of a given amount of resource is how much the user is willing to pay for that quantity. The inverse of the valuation is the user's demand function, giving a desired quantity for each price.

her (economic) efficiency objectives, and then – if necessary – transform it into an equivalent mechanism in the desired message space. This is convenient because one can exclude the infinitely many mechanisms with larger message spaces, without fear of missing any better designs. The design process is usually the solution of an optimization (mathematical programming) problem. For this reason, in the literature, mechanism design problems are mostly solved for cases where the space of users' types is one dimensional, or at most finite dimensional [21], using message spaces that are of the same dimension.

In our sharing problem, the conventional approach is unsatisfactory in two ways:

- First, a user's type is infinite-dimensional, as we do not restrict the valuation functions beyond some very general assumptions (see Section 3.1), and so the conventional "programming" approach of deriving the mechanism from the revelation principle would lead to an intractable problem.
- Second, the conventional (direct revelation) approach, even if it was tractable, implies that a single message (bid) can theoretically be infinitely long, because it has to contain a description of the function  $\theta_i$ . Clearly, this is not desirable in a communication network, where signaling load is a key consideration. For engineering reasons, we choose a message space that is 2-dimensional. Therefore, a given message can come from many possible types, so there is no single way to do the transformation from the direct revelation mechanism to the desired one.

Thus, unlike most of the mechanism design literature, we will take a direct approach, where we posit an allocation rule for our desired message space, and then show that it has an equilibrium, and that the design objective is met at equilibrium<sup>4</sup>. This is equivalent to guessing the right direct-revelation-to-desired-mechanism transformation and building it into the allocation rule from the start.

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<sup>4</sup>Our aim is to show that we can use this smaller message space and still achieve our objective.

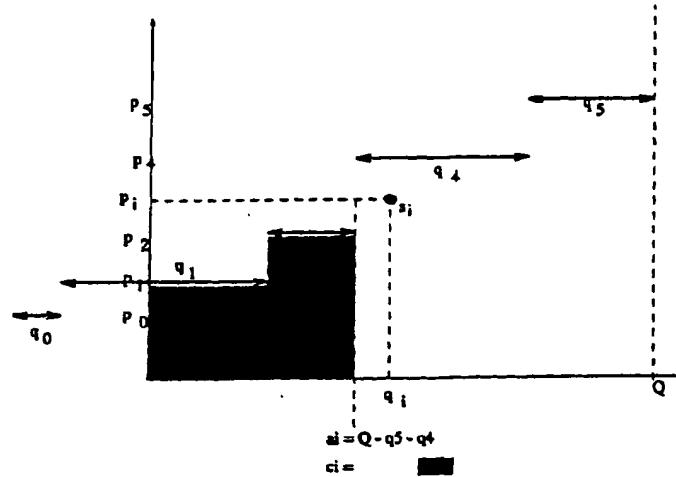


Figure 1: Exclusion-compensation principle: the intuition behind the PSP rule

## 2.2 Allocation Rule

Define, for  $y \geq 0$

$$\underline{Q}_i(y, s_{-i}) = \left[ Q - \sum_{p_k \geq y, k \neq i} q_k \right]^+. \quad (1)$$

and

$$Q_i(y, s_{-i}) = \lim_{\eta \searrow y} \underline{Q}_i(\eta, s_{-i}) = \left[ Q - \sum_{p_k > y, k \neq i} q_k \right]^+.$$

The "progressive second price" (PSP) allocation rule is defined as follows:

$$a_i(s) = q_i \wedge \underline{Q}_i(p_i, s_{-i}), \quad (2)$$

$$c_i(s) = \sum_{j \neq i} p_j [a_j(0; s_{-i}) - a_j(s_i; s_{-i})], \quad (3)$$

where  $\wedge$  means taking the minimum.

**Remark a:** For a fixed opponent profile  $s_{-i}$ ,  $Q_i(p_i, s_{-i})$  represents the maximum available quantity at a bid price of  $p_i$ . The intuition behind PSP is an exclusion-compensation principle: player  $i$  pays for his allocation

so as to exactly cover the "social opportunity cost" which is given by the declared willingness to pay (bids) of the users who are excluded by  $i$ 's presence (see Figure 1), and thus also compensates the seller for the maximum lost potential revenue. Note that this amounts to implicitly assuming that the bid price accurately reflects the marginal valuation  $\theta_i^t$  on the range  $[a_i, q_i]$ . In other words, by this rule the auctioneer is saying to the player: "if you bid  $(q_i, p_i)$ , I take it to mean that in the vicinity of  $q_i$ ,  $\theta_i$  can be approximated by a line of slope  $p_i$ ." This is the (built-in) transformation from the direct-revelation mechanism to the desired message process discussed in the second remark at the end of Section 2.1.

The charge  $c_i$  increases with  $a_i$  in a manner similar to the income tax in a progressive tax system. For a fixed opponent profile  $s_{-i}$ , imagine player  $i$  is increasing  $q_i$ , starting from 0. The first few units that player  $i$  gets will be taken away from the lowest clearing opponent (i.e.  $m = \arg \min_j \{p_j : a_j > 0\}$ ), and player  $i$  will pay a price (marginal cost)  $p_m$  per unit. When  $a_m$  reaches 0, the subsequent units that player  $i$  gets will cost him  $p_{m'} > p_m$ , where  $m'$  is the new lowest clearing player, the one just above  $m$ . The PSP rule is the natural generalization of second-price auctions (or Vickrey auctions). In a Vickrey auction of a *single non-divisible object*, each player submits a sealed bid, and the object is sold to the highest bidder at the bid price of the second highest bidder, which is what happens here if  $q_i = Q, \forall i$ . This is widely known to have many desirable properties [35, 24, 4], the most important of which is that it has an equilibrium profile where all players bid their true valuation. As we will presently show, this property is preserved by the PSP rule in the more general case of sharing an arbitrarily divisible resource, and this leads to stability (Nash equilibrium). The PSP rule is analogous to Clarke-Groves mechanisms [3, 8, 20] in the direct-revelation case.

**Remark b:** When two players bid at exactly the same price, and they are asking for more than is available at that price, (2) punishes both of them. For example, if  $Q = 100$  and  $s_1 = (4, 60)$  and  $s_2 = (4, 70)$ , the allocations would be  $a_1 = 60 \wedge (100 - 70) = 30$ , and  $a_2 = 70 \wedge (100 - 60) = 40$ . Since the bid prices are equal, there is no "right" way to decide who to give the remaining capacity to. One could divide it equally, or proportionally to their requests, etc. For the subsequent analysis, it turns out it is simpler to not give it to either one (of course, it will be allocated to the lower bidders if there are any). This is just a technicality since by deciding this, we ensure that it will never happen (at equilibrium), since the users will always prefer to change their prices and/or reduce their quantity.

Considering the computational complexity of PSP, a straightforward implementation would at worst, sort the bids in time  $I \log I$ , perform (2) in linear time, and (3) can be done in time  $I^2$ . Thus, the complexity of computing the allocations is  $O(I^2)$ .

### 3 Analysis of the Progressive Second Price Auction

#### 3.1 User Preferences

Since the allocation rule  $A$  is given by design, the only analytical assumptions we make is on the form of the players' preferences.

Player  $i$ 's preferences are given by his utility function

$$\begin{aligned} u_i : S &\longrightarrow (-\infty, \infty) \\ s &\longmapsto u_i(s). \end{aligned}$$

Player  $i$  has a valuation of the resource  $\theta_i(a_i(s)) \geq 0$ , which is the total value to her of her allocation. Thus, for a bid profile of  $s$ , under allocation rule  $A$ , player  $i$  getting an allocation  $A_i(s)$  has the quasi-linear utility

$$u_i(s) = \theta_i(a_i(s)) - c_i(s) \quad (4)$$

which is simply the value of what she gets minus the cost.

In addition, the player can be constrained by a budget  $b_i \in [0, \infty]$ , so the bid  $s_i$  must lie in the set

$$S_i(s_{-i}) = \{s_i \in S_i : c_i(s_i; s_{-i}) \leq b_i\}. \quad (5)$$

In the proofs of the following section, we will assume that users have elastic demand, that is:

**Assumption 1** For any  $i \in \mathcal{I}$ ,

- $\theta_i(0) = 0$ .
- $\theta_i$  is differentiable,
- $\theta'_i \geq 0$ , non-increasing and continuous
- $\exists \gamma_i > 0$ .  $\forall z \geq 0$ ,  $\theta'_i(z) > 0 \Rightarrow \forall \eta < z$ ,  $\theta'_i(z) \leq \theta'_i(\eta) - \gamma_i(z - \eta)$ .

The last item says that as long as the valuation is strictly increasing, it must also be strictly concave (with minimum curvature  $\gamma_i$ ). However, it is allowed to "flatten" beyond a certain amount of resource.

Functions of this (concave) form have wide applicability as models of resource valuation, and can be justified from the economic standpoint (diminishing returns) as well as from information theoretic standpoint – see Appendix A. For examples of valuations satisfying Assumption 1, see Section 3.4 and Appendix A.

### 3.2 Equilibrium of PSP

The auction game is given by  $(Q, u_1, \dots, u_I, A)$ , that is, by specifying the resource, the players, and a feasible allocation rule. We analyze it as a strategic game of complete information [4].

Define the set of best replies to a profile  $s_{-i}$  of opponents bids:  $S_i^*(s_{-i}) = \{s_i \in S_i(s_{-i}) : u_i(s_i; s_{-i}) \geq u_i(s'_i; s_{-i}), \forall s'_i \in S_i(s_{-i})\}$ . Let  $S^*(s) = \prod_i S_i^*(s_{-i})$ . A Nash equilibrium is a fixed point of the point-to-set mapping  $S^*$ , i.e. a profile  $s \in S^*(s)$ . In other words, it is a point from which no player will want to unilaterally deviate. Such a point is what is most accepted as a consistent prediction of the actual outcome of a game, and has been repeatedly confirmed by experiments, as well as a wide range of theoretical approaches. Indeed, in a dynamic game, where players recompute the best response to the current strategy profile of their opponents, this iteration can only converge to a Nash equilibrium (if it converges at all). In addition, an important trend in modern game theory is the development of learning models, and there too, it has been shown that Nash equilibria result also from rational learning through repeated play among the same players [13].

A more general (and hence weaker) notion of stability is the existence of an  $\epsilon$ -Nash equilibrium. Let the  $\epsilon$ -best replies be  $S_i^\epsilon(s_{-i}) = \{s_i \in S_i(s_{-i}) : u_i(s_i; s_{-i}) \geq u_i(s'_i; s_{-i}) - \epsilon, \forall s'_i \in S_i(s_{-i})\}$ . An  $\epsilon$ -Nash equilibrium is a fixed point of  $S^\epsilon$ .

In a dynamic auction game,  $\epsilon > 0$  can be interpreted as a *bid fee* paid by a bidder each time they submit a bid. Thus, the user will send a best reply bid as long as it improves her current utility by  $\epsilon$ , and the game can only end at an  $\epsilon$ -Nash equilibrium.

Define

$$P_i(z, s_{-i}) = \inf \{y \geq 0 : Q_i(y, s_{-i}) \geq z\}. \quad (6)$$

Thus, for fixed  $s_{-i}$ ,  $\forall y, z \geq 0$ ,

$$z \leq Q_i(y, s_{-i}) \Rightarrow y \geq P_i(z, s_{-i}) \quad (7)$$

and<sup>5</sup>

$$y > P_i(z, s_{-i}) \Rightarrow z \leq Q_i(y, s_{-i}). \quad (8)$$

The graph of  $P_i(\cdot, s_{-i})$  is the "staircase" shown in Figure 1, and that of  $Q_i(\cdot, s_{-i})$  is obtained by flipping it 90 degrees.

It is readily apparent that

$$c_i(s) = \int_0^{a_i(s)} P_i(z, s_{-i}) dz. \quad (9)$$

The key property of PSP is that, for a given opponent profile, a player cannot do much better than simply tell the truth, which in this setting means bidding at a price equal to the marginal valuation, i.e. set  $p_i = \theta'_i(q_i)$ . By doing so, she can always get within  $\epsilon > 0$  of the best utility.

Let  $T_i = \{s_i \in S_i : p_i = \theta'_i(q_i)\}$ , the (unconstrained) set of player  $i$ 's truthful bids, and  $T = \prod_i T_i$ .

**Proposition 1** (Incentive compatibility) *Under Assumption 1,  $\forall i \in I$ ,  $\forall s_{-i} \in S_{-i}$ , such that  $Q_i(0, s_{-i}) = 0$ , for any  $\epsilon > 0$ , there exists a truthful  $\epsilon$ -best reply  $t_i(s_{-i}) \in T_i \cap S_i^\epsilon(s_{-i})$ .*

In particular, let

$$G_i(s_{-i}) = \left\{ z \in [0, Q] : z \leq Q_i(\theta'_i(z), s_{-i}) \text{ and } \int_0^z P_i(\eta, s_{-i}) d\eta \leq b_i \right\}.$$

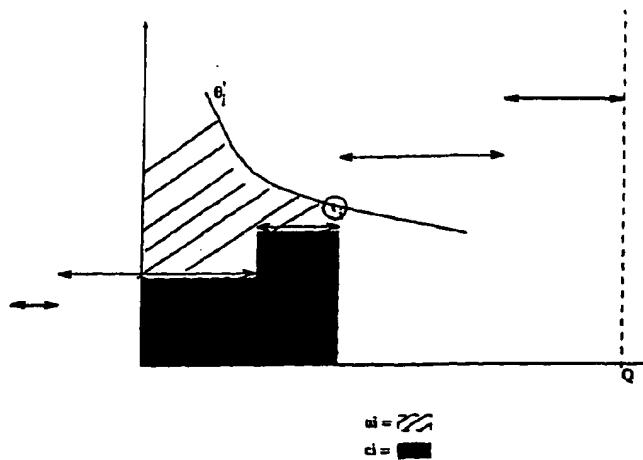
Then with  $v_i = [\sup G_i(s_{-i}) - \epsilon/\theta'_i(0)]^+$  and  $w_i = \theta'_i(v_i)$ ,  $t_i = (v_i, w_i) \in T_i \cap S_i^\epsilon(s_{-i})$ .

The truthful best reply can be found in a straightforward manner, as illustrated in Figure 2.

**Proof:** Fix  $s_{-i} \in S_{-i}$ . Let  $z_i = \sup G_i(s_{-i})$  and  $y_i = \theta'_i(z_i)$ .

By definition of  $z_i$ ,  $\exists \{z(n)\} \subset G_i(s_{-i})$  such that  $\lim_n z(n) = z_i$ . Hence  $b_i \geq \lim_n \int_0^{z(n)} P_i(\eta, s_{-i}) d\eta = \int_0^{z_i} P_i(\eta, s_{-i}) d\eta \geq c_i(t_i; s_{-i})$ , where the equality comes from the boundedness of  $P_i$  and the Lebesgue dominated convergence theorem, and the second inequality from (9) and (2). Thus  $t_i \in T \cap S_i(s_{-i})$ .

<sup>5</sup>Actually, since  $Q_i(\cdot, s_{-i})$  is upper-semi-continuous (jumps up), we have  $z \leq Q_i(y, s_{-i}) - y \geq P_i(z, s_{-i})$

Figure 2: Truthful  $\epsilon$ -best reply

Next we show that  $t_i \in S_i^*(s_{-i})$ . First,  $z_i = \lim_n z(n) \leq \lim_n Q_i(\theta'_i(z(n)), s_{-i}) \leq Q_i(\lim_n \theta'_i(z(n)), s_{-i})$ , where the inequalities follow respectively from  $z(n) \in G_i(s_{-i})$ , and the upper semi-continuity of  $Q_i(., s_{-i})$ . Now by the continuity of  $\theta'_i$ ,  $Q_i(\lim_n \theta'_i(z(n)), s_{-i}) = Q_i(\theta'_i(z_i), s_{-i}) = Q_i(y_i, s_{-i})$ , hence

$$z_i \leq Q_i(y_i, s_{-i}). \quad (10)$$

Now, we claim that  $a_i(t_i; s_{-i}) = v_i$ . Indeed, if  $z_i = 0$  then  $v_i = 0$  and  $a_i(t_i; s_{-i}) = 0$ . If  $z_i > 0$ , then by (10),  $Q_i(y_i, s_{-i}) > 0$  and since by hypothesis  $Q_i(0, s_{-i}) = 0$ , we have  $\theta'_i(z_i) = y_i > 0$ . Also,  $z_i > 0$  implies  $v_i < z_i$ . Therefore, by Assumption 1, we have  $w_i = \theta'_i(v_i) > \theta'_i(z_i) = y_i$ . Hence, since  $Q_i(., s_{-i})$  is non-decreasing,  $Q_i(w_i, s_{-i}) \geq \lim_{\eta \searrow y} Q_i(\eta, s_{-i}) = Q_i(y_i, s_{-i}) \geq z_i > v_i$ . Thus, by (2),

$$a_i(t_i; s_{-i}) = v_i \quad (11)$$

Now  $\forall s_i \in S_i(s_{-i})$ ,

$$\begin{aligned} u_i(t_i; s_{-i}) - u_i(s) &= \theta_i(a_i(t_i; s_{-i})) - \theta_i(a_i(s)) - c_i(t_i; s_{-i}) + c_i(s) \\ &= \int_{a_i(t_i; s_{-i})}^{a_i(s)} [P_i(z, s_{-i}) - \theta'_i(z)] dz. \\ &= \int_{z_i}^{a_i(s)} [P_i(z, s_{-i}) - \theta'_i(z)] dz + \int_{v_i}^{x_i} [P_i(z, s_{-i}) - \theta'_i(z)] dz \end{aligned}$$

$$\geq \int_{z_i}^{a_i(s)} [P_i(z, s_{-i}) - \theta'_i(z)] dz - \epsilon \quad (12)$$

where the inequality follows from  $(z_i - v_i) \leq \epsilon/\theta'_i(0)$  and the fact that  $\theta'_i$  is non-increasing. Thus, it suffices to show that the integral is  $\geq 0$ .

If  $z_i < a_i(s)$ , take any  $z \in (z_i, a_i(s))$ . By the definition of  $z_i$ ,  $z \notin G_i(s_{-i})$ . Now  $s_i \in S_i(s_{-i})$  implies  $b_i \geq c_i(s) = \int_0^{a_i(s)} P_i(\eta, s_{-i}) d\eta \geq \int_0^z P_i(\eta, s_{-i}) d\eta$ . Therefore, we must have  $z > Q_i(\theta'_i(z))$ , which by (8), implies  $\theta'_i(z) \leq P_i(z)$  and the integrand in (12) is  $\geq 0$  as desired.

Suppose  $z_i \geq a_i(s_i)$ . Since  $\theta'_i$  is non-increasing,  $Q_i(\cdot, s_{-i})$  is non-decreasing and  $P_i(\cdot, s_{-i}) \geq 0$ , any point to the left of  $z_i$  is in the set  $G_i(s_{-i})$ ,  $\forall z < z_i, z \in G_i(s_{-i})$ , hence  $z \leq Q_i(\theta'_i(z), s_{-i})$  which by (7), implies  $\theta'_i(z) \geq P_i(z, s_{-i})$ , so the integrand in (12) is  $\leq 0$  as desired.  $\square$

Figure 3 shows the utility function of player 4,  $u_4(s_4)$ , in a PSP auction with  $I = 5$  players, with  $s_{-4}$  fixed, and a valuation  $\theta_4(q) = 10q$ . The plateaus correspond to the points where  $q_4 \geq Q_4(p_4, s) = [Q - \sum_{j:p_j > p_4} a_j(s)]^+$ , and  $a_4(s)$  can no longer be increased at that bid price – see (2). At bid prices  $p_4 > p_5$ , the utility decreases when  $a_4 > Q - q_5$ , because after that point, each additional unit of resource is taken away from player 5, and thus costs  $p_5$ , which is more than  $\theta'_4$  its value to player 4. Thus, each additional unit starts bringing negative utility. This is what discourages users from bidding above their valuation. Proposition 1 is illustrated by the fact that for any given quantity  $q_4$ , the utility  $u_4$  is maximized on the plane  $p_4 = \theta'_4 \equiv 10$ .

**Remark:** When the players have linear valuations and no budget constraint ( $b_i = \infty$ ), PSP becomes identical to a second-price auction for a non-divisible object. Then the existence of a Nash equilibrium follows directly from incentive compatibility.

Note that in PSP, the incentive compatibility (optimality of truth-telling) is in the price dimension, for a given quantity. With the message space we have designed, there is no single “true” quantity to declare, the optimal quantity depends on opponent bid prices. Were the message process such that players declared a price and a budget (rather than desired quantity), it may have been possible to design an allocation rule  $A$  such that they are inclined to reveal their true budget, thus obtaining incentive compatibility in both dimensions, and hence equilibrium. But such a rule  $A$  would likely not have a simple closed form like (2) and (3). In essence, the computational load of translating budgets into shares would be centralized at the auctioneer, thus making the system less scalable to large numbers of users. On the

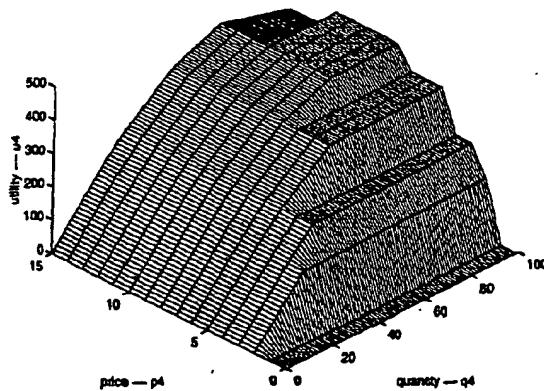


Figure 3: Utility  $u_4(s_4)$  for  $s_1 = (100, 1)$ ,  $s_2 = (10, 2)$ ,  $s_3 = (20, 4)$ ,  $s_5 = (20, 7)$ ,  $s_6 = (30, 12)$

other hand, decentralization has a cost too, which is the signaling overhead resulting from players possibly adjusting bids based on opponent bids in the iterated game. Our design is based on the premise that the latter approach is the more scalable of the two (indeed that was the reason for choosing a small message space).

The next property is that the truthful best reply is continuous in opponent profiles (this can be seen in Figure 2: as the “staircase” is varied smoothly, the point of intersection with  $\theta_i'$  moves smoothly, provided  $\theta_i'$  is not flat – which is given by the last time item in Assumption 1). To prove that, we will need the following:

**Lemma 1**  $\forall s, s' \in S, \forall y, z \geq 0, \forall \delta > 0$ , if  $\|s_{-i} - s'_{-i}\| < \delta$  then

$$Q_i(y + \delta, s_{-i}) + \delta\sqrt{I} \geq Q_i(y, s'_{-i}) \geq Q_i(y - \delta, s_{-i}) - \delta\sqrt{I}, \quad (13)$$

and

$$P_i(z + \delta\sqrt{I}, s_{-i}) + \delta \geq P_i(z, s'_{-i}) \geq P_i(z - \delta\sqrt{I}, s_{-i}) - \delta. \quad (14)$$

**Proof:** First,  $\|s_{-i} - s'_{-i}\| < \delta$  implies  $\sum_k |q'_k - q_k| < \delta\sqrt{I}$ , and  $p_k + \delta > p'_k > p_k - \delta$ . Thus,  $\sum_k q_k l_{\{p_k + \delta > y\}} + \delta\sqrt{I} \geq \sum_k q'_k l_{\{p'_k > y\}} \geq \sum_k q_k l_{\{p_k - \delta > y\}} - \delta\sqrt{I}$ . Then, using (1) and the identity  $(a+b)^+ \leq (a)^+ + (b)^+$ , the first result follows.

For any  $y < P_i(z, s'_{-i})$ , by (7), we have  $z > Q_i(y, s'_{-i}) \geq Q_i(y - \delta, s_{-i}) - \delta\sqrt{I}$ , which by (8), implies  $y - \delta \leq P_i(z + \delta\sqrt{I}, s_{-i})$ . Letting  $y \nearrow P_i(z, s'_{-i})$ , we get  $P_i(z, s'_{-i}) \leq P_i(z + \delta\sqrt{I}, s_{-i}) + \delta$ .

For any  $y > P_i(z, s'_{-i})$ , by (8), we have  $z \leq Q_i(y, s'_{-i}) \leq Q_i(y + \delta, s_{-i}) + \delta\sqrt{I}$ , which by (7) implies  $y + \delta \geq P_i(z - \delta\sqrt{I})$ . Letting  $y \searrow P_i(z, s'_{-i})$ , we get  $P_i(z, s'_{-i}) \geq P_i(z - \delta\sqrt{I}) - \delta$ .  $\square$

**Lemma 2 (Continuity of best reply)** Under Assumption 1,  $\forall i \in I$ , the  $\epsilon$ -best reply  $t_i$  given in Proposition 1 is continuous in  $s_{-i}$  on any subset  $V_i(\underline{P}, \bar{P}) = \{s_{-i} \in S_i : \forall z > 0, \bar{P} \geq P_i(z, s_{-i}) \geq \underline{P}\}$ , with  $\infty > \bar{P} \geq \underline{P} > 0$ .

**Proof:** Let  $z_i = \sup G_i(s_{-i})$ . We will show  $z_i$  is continuous, and the continuity of  $v_i = [z_i - \epsilon/\theta'_i(0)]^+$  and  $w_i = \theta'_i(v_i)$  follow immediately (recall that by Assumption 1,  $\theta'_i$  is continuous).

Suppose there is a discontinuity at some  $s_{-i}$ . Then,  $\exists \epsilon_1 > 0$ , such that  $\forall \delta > 0$ ,  $\exists s'_{-i} \in V_i(\underline{P}, \bar{P})$  with  $\|s_{-i} - s'_{-i}\| < \delta$  and  $\|z_i - z'_i\| \geq \epsilon_1$ , where  $z'_i = \sup G_i(s'_{-i})$ .

Suppose  $z_i + \epsilon_1 \leq z'_i = \sup G_i(s'_{-i})$  (the case  $z'_i + \epsilon_1 \leq z_i$  is handled identically, with  $s_{-i}$  and  $s'_{-i}$  interchanged.). Consider the definition of  $G_i$ ; since  $\theta'_i$  is decreasing and  $Q_i(\cdot, s_{-i})$  is non-decreasing and  $P_i(\cdot, s_{-i}) \geq 0$ , any point to the left of  $z'_i$  is in the set  $G_i(s'_{-i})$ , therefore

$$z_i + \epsilon_1 \in G_i(s'_{-i}). \quad (15)$$

Thus,  $z_i + \epsilon_1 \leq Q_i(\theta'_i(z_i + \epsilon_1), s'_{-i}) = \left[ Q - \sum_k q'_k 1_{\{p'_k > \theta'_i(z_i + \epsilon_1)\}} \right]^+$ . Therefore,

$$z_i + \epsilon_1 \leq Q_i(\theta'_i(z_i + \epsilon_1) + \delta, s_{-i}) + \delta\sqrt{I},$$

using Lemma 1.

Also, by (7)  $z_i + \epsilon_1 \leq Q_i(\theta'_i(z_i + \epsilon_1), s'_{-i}) \Rightarrow \theta'_i(z_i + \epsilon_1) > P(z_i + \epsilon_1, s'_{-i})$ . Now since  $s'_{-i} \in V_i(\underline{P}, \bar{P})$ , this last expression is  $\geq \underline{P} > 0$ , hence  $\theta'_i(z_i + \epsilon_1) > 0$ . Then using Assumption 1,  $\theta'_i(z_i + \epsilon_2) \geq \theta'_i(z_i + \epsilon_1) + \gamma_i(\epsilon_1 - \epsilon_2) > \theta'_i(z_i + \epsilon_1) + \delta$ , for  $\delta < \delta_1 = \frac{\epsilon_1}{\sqrt{I}} \wedge \epsilon_1 \gamma_i$ , and  $0 < \epsilon_2 < (\epsilon_1 - \delta_1 \sqrt{I}) \wedge (\epsilon_1 - \delta_1 / \gamma_i)$ . Therefore, since  $Q_i(\cdot, s_{-i})$  is non-decreasing,

$$\begin{aligned} z_i + \epsilon_2 &\leq Q_i(\theta'_i(z_i + \epsilon_2), s_{-i}) + \delta\sqrt{I} - \epsilon_1 + \epsilon_2 \\ &< Q_i(\theta'_i(z_i + \epsilon_2), s_{-i}). \end{aligned} \quad (16)$$

Now (15) also implies that

$$\begin{aligned} b_i &\geq \int_0^{z_i + \epsilon_1} P_i(\eta, s'_{-i}) d\eta \\ &\geq \int_0^{z_i + \epsilon_2} P_i(\eta, s_{-i}) d\eta + \int_0^{z_i + \epsilon_2} [P_i(\eta, s'_{-i}) - P_i(\eta, s_{-i})] d\eta + (\epsilon_1 - \epsilon_2) \underline{P}. \end{aligned}$$

and this holds  $\forall \epsilon_3 < \epsilon_1$ . Now, using Lemma 1,

$$\begin{aligned} & \int_0^{z_i+\epsilon_3} [P_i(\eta, s_{-i}) - P_i(\eta, s_{-i})] d\eta \\ & \geq -\delta Q + \int_0^{z_i+\epsilon_3} P_i(\eta - \delta\sqrt{I}, s_{-i}) d\eta - \int_0^{z_i+\epsilon_3} P_i(\eta, s_{-i}) d\eta \\ & \geq -\delta Q - \int_{z_i+\epsilon_3-\delta\sqrt{I}}^{z_i+\epsilon_3} P_i(\eta, s_{-i}) d\eta \\ & \geq -(Q + \bar{P})\delta\sqrt{I}. \end{aligned}$$

Let  $\delta_2 = \frac{\epsilon_1 P}{(Q+P)\sqrt{I}}$ , and  $\epsilon_3$  such that  $0 < \epsilon_3 < \epsilon_1 - (Q + \bar{P})\delta_2\sqrt{I}/P$ . Then

$$b_i \geq \int_0^{z_i+\epsilon_3} P_i(\eta, s_{-i}) d\eta, \quad (17)$$

for  $\delta < \delta_2$ .

Now choosing  $\delta < \delta_1 \wedge \delta_2$ , (16) and (17) imply that  $G_i(s_{-i}) \ni (z_i + \epsilon_3) \wedge (z_i + \epsilon_2) > z_i = \sup G_i(s_{-i})$ , a contradiction.  $\square$

We introduce one additional player, player 0, whose valuation is  $\theta_0(z) = p_0 z$ , and whose bid can therefore be fixed at  $s_0 = (q_0, p_0) = (Q, p_0)$ . Player 0 can be viewed as the auctioneer, and  $p_0 > 0$  as a "reserve price" at which the seller is willing to "buy" all of the resource from himself. From (1), the presence of the bid  $s_0 = (Q, p_0)$  implies  $\forall i \in \mathcal{I}, Q_i(y, s_{-i}) = 0, \forall y < p_0$ . In particular, setting  $y = 0$ , the condition of Proposition 1 holds. Thus, we can restrict our attention to truthful strategies only, and still have feasible best replies. This forms a "truthful" game embedded within the larger auction game, where the strategy space is  $\mathcal{T} \subset \mathcal{S}$ , the feasible sets are  $\mathcal{T}_i \cap S_i(s_{-i})$ , and the best replies are  $R_i^*(s) = \mathcal{T}_i \cap S_i^*(s)$ . A fixed point of  $R^*$  in  $\mathcal{T}$  is a fixed point of  $S^*$  in  $\mathcal{S}$ . Thus an equilibrium of the embedded game is an equilibrium of the whole game.

**Proposition 2 (Nash equilibrium)** *In the auction game with the PSP rule given by (2) and (3) and a reserve price  $p_0 > 0$ , and players described by (4) and (5), if Assumption 1 holds, then for any  $\epsilon > 0$ , there exists a truthful  $\epsilon$ -Nash equilibrium  $s^* \in \mathcal{T}$ .*

**Proof:**  $\forall s \in \mathcal{T}, \forall i \in \mathcal{I}, \forall z > 0$ , we have  $z > 0 = Q_i(p_0/2, s_{-i})$ , which by (8) implies  $P_i(z, s_{-i}) \geq p_0/2 = P$ . Let  $\bar{P} = \max_{k \in \mathcal{K} \cup \{0\}} \theta_k'(0)$ . Then, the conditions of Lemma 2 are satisfied and  $t = (v, w)$  is continuous in  $s$  on  $\mathcal{T}$ . By Assumption 1,  $\theta'_i$  is continuous therefore  $v(q, p) = v(q, \theta'(q))$  (as defined in Proposition 1),

can be viewed as a continuous mapping of  $[0, Q]^I$  onto itself. By Brouwer's fixed-point theorem (see for example [10]), any continuous mapping of a convex compact set into itself has at least one fixed point, i.e.  $\exists q^* = v(q^*) \in [0, Q]^I$ . Now with  $s^* = (q^*, \theta'(q^*))$ , we have  $s^* = t(s^*) \in T$ .  $\square$

### 3.3 Efficiency

The objective in designing the auction is that, at equilibrium, resources always go to those who value them most. Indeed, the PSP mechanism does have that property. This can be loosely argued as follows: for each player, the marginal valuation is never greater than the bid price of any opponent who is getting a non-zero allocation. Thus, whenever there is a player  $j$  whose marginal valuation is less than player  $i$ 's and  $j$  is getting a non-zero allocation,  $i$  can take some away from  $j$ , paying a price less than  $i$ 's marginal valuation, i.e. increasing  $u_i$ , but also increasing the total value, since  $i$ 's marginal value is greater. Thus at equilibrium, i.e. when no one can unilaterally increase their utility, the total value is maximized. Formally, consider  $a \in \arg \max_A \sum_i \theta_i(a_i)$ . The Karush-Kuhn-Tucker [17] optimality conditions are that there exists a Lagrange multiplier  $\lambda$  such that  $\theta'_i(a_i) = \lambda$ , if  $a_i > 0$ , and  $\theta'_i(0) \leq \lambda$ , if  $a_i = 0$ .

**Assumption 2** For any  $i \in I$ ,  $b_i = \infty$ , and  $\theta'_i$  satisfies<sup>6</sup>

$$\theta'_i(z) - \theta'_i(z') > -\kappa(z - z'),$$

whenever  $z > z' \geq 0$ .

Given any  $\epsilon > 0$ , for any  $\epsilon$ -Nash-equilibrium  $s^* \in T$ , let  $a^* \equiv a(s^*)$ , and let  $\underline{a}^* \equiv \min_{i \in I \cup \{0\}, a_i > 0} a_i^*$ , the smallest non-zero allocation. The following is the " $\epsilon$  version" of the Karush-Kuhn-Tucker conditions.

**Lemma 3** Suppose Assumptions 1 and 2 hold. If for some  $j$ ,  $a_j^* > \sqrt{\epsilon/\kappa}$ , then  $\forall i \in I \cup \{0\}$ ,

$$\theta'_i(a_i^*) < \theta'_j(a_j^*) + 2\sqrt{\epsilon\kappa}.$$

An immediate corollary is that if  $\underline{a}^* > \sqrt{\epsilon/\kappa}$  then

$$\lambda^* - 2\sqrt{\epsilon\kappa} < \theta'_i(a_i^*) < \lambda^* + 2\sqrt{\epsilon\kappa}$$

<sup>6</sup>If  $\theta'_i$  is differentiable, the condition is  $0 \geq \theta''_i > -\kappa$ .

if  $a_i^* > \sqrt{\epsilon/\kappa}$  and

$$\theta'_i(a_i^*) < \lambda^* + 2\sqrt{\epsilon\kappa},$$

if  $a_i^* = 0$ , for some  $\lambda^* \geq 0$ .

**Proof:** Suppose  $\theta'_i(a_i^*) \geq \theta'_j(a_j^*) + 2\sqrt{\epsilon\kappa}$ . Since  $\theta'_j(a_j^*) \geq \theta'_j(q_j^*) \geq p_j^*$ , we have  $\theta'_i(a_i^*) \geq p_j^* + 2\sqrt{\epsilon\kappa}$ .

Since  $a_j^* > 0$ , if player  $i$  bids at a price above  $p_j^*$ , he can take all of player  $j$ 's allocation, without losing anything of his own, i.e.  $a_i^* + a_j^* \leq Q_i(p_j^*, s_{-i}^*)$ . By (7), this implies

$$p_j^* \geq P_i(a_i^* + a_j^*, s_{-i}^*).$$

Let  $q_i = (a_i^* + \sqrt{\epsilon/\kappa})$  and  $s_i = (q_i, \theta'_i(q_i))$ . Then

$$\begin{aligned} u_i(s_i; s_{-i}^*) - u_i(s^*) &= \int_{a_i^*}^{a_i^* + \sqrt{\epsilon/\kappa}} \theta'_i(z) - P_i(z, s_{-i}^*) dz \\ &\geq [\theta'_i(a_i^* + \sqrt{\epsilon/\kappa}) - p_j^*] \sqrt{\epsilon/\kappa} \\ &\geq [\theta'_i(a_i^*) - \kappa\sqrt{\epsilon/\kappa} - p_j^*] \sqrt{\epsilon/\kappa} \\ &\geq [2\sqrt{\epsilon\kappa} - \kappa\sqrt{\epsilon/\kappa}] \sqrt{\epsilon/\kappa} \\ &= \epsilon \end{aligned}$$

which contradicts the fact that  $s^*$  is an  $\epsilon$ -Nash equilibrium.  $\square$

**Proposition 3 (Efficiency)** Suppose Assumptions 1 and 2 hold. If  $a_i^* > \sqrt{\epsilon/\kappa}$ , then

$$\max_A \sum_i \theta_i(a_i) - \sum_i \theta_i(a_i^*) = O(\sqrt{\epsilon\kappa}),$$

where  $A = \{a \in [0, Q]^{I+1} : \sum_i a_i \leq Q\}$ .

**Proof:** (of Proposition 3) Let  $I^+ = \{k : a_k > a_k^*\}$  and  $I^- = \{k : a_k < a_k^*\}$ . For  $i \in I^+$ , we have  $\theta'_i(a_i^*) \leq \lambda^* + 2\sqrt{\epsilon\kappa}$ . For  $i \in I^-$ , we have  $a_i^* > a_i \geq 0$ , therefore by the lemma,  $\theta'_i(a_i^*) > \lambda^* - 2\sqrt{\epsilon\kappa}$ . Therefore,

$$\begin{aligned} \sum_I \theta_i(a_i) - \theta_i(a_i^*) &\leq \sum_{I^+} \theta'_i(a_i^*)(a_i - a_i^*) - \sum_{I^-} \theta'_i(a_i^*)(a_i^* - a_i) \\ &\leq (\lambda^* + 2\sqrt{\epsilon\kappa})\Delta - (\lambda^* - 2\sqrt{\epsilon\kappa})\Delta, \end{aligned}$$

where  $\Delta = \sum_{I^+}(a_i - a_i^*) = \sum_{I^-}(a_i^* - a_i)$ . Since  $\Delta \leq Q$  the result follows, with the bound  $4Q\sqrt{\epsilon\kappa}$ .  $\square$

**Remark a:** The condition  $b_i = \infty$  is sufficient, but not necessary to achieve efficient outcomes. In fact with any budget profile, efficiency can be achieved if the users cooperate. For example, if they all choose a bid quantity close to what they can actually obtain (which they do if they use the strategy given by Proposition 1), then the price paid would be  $p_0$  per unit for all the allocations, and if  $p_0$  or the shares  $a_i^*$  are not too large, then budget constraints are irrelevant and  $a^*$  is efficient. More generally, efficiency is attained if the budgets are not too far out of line with the valuations. i.e. there are no players with very high demand and very low budget.

**Remark b:** (*Welfare and Efficiency*) A more common measure of efficiency is the social welfare, which is the sum of all the players' utility  $\sum_i u_i$ , including the seller  $i = 0$ . The natural definition of the seller's utility is the value of the leftover capacity  $a_0 = Q - \sum_{i \neq 0} a_i$  plus the revenue, i.e.

$$u_0 = \theta_0(a_0) + \sum_{i \neq 0} c_i.$$

Then,  $\sum_i u_i = \sum_{i \neq 0} (\theta_i - c_i) + u_0 = \sum_{i \neq 0} \theta_i + \theta_0(a_0) = \sum_i \theta_i$ . Thus,  $\sum_i u_i$  is equivalent to the efficiency measure used above, which is  $\sum_i \theta_i$ . Another measure is the seller's revenue. Even though PSP is not, in general, revenue-maximizing, it tends to the revenue maximizing allocations and prices as demand increases [18].

**Remark c:** Proposition 3 provides a key to understanding the basic trade-off between engineering and economic efficiency. The smaller  $\epsilon$ , the closer we get to the value-optimal allocations. But in a dynamic game, where players iteratively adjust their bids to the opponent profile, a player will bid as long as he can gain at least  $\epsilon$  utility (since that is the cost of the bid), thus a smaller  $\epsilon$  makes the iteration take longer to converge, i.e. entails more signaling.

### 3.4 Convergence

An issue of obvious concern is whether the game converges under dynamic play: it turns out that it does, when users behave rationally (see Proposition 4 in Chapter 2 of [29]). Moreover, irrational or malicious behavior – like intentionally trying to prevent convergence by making unnecessary bids – can always be controlled by setting the bid fee  $\epsilon$  high enough to make such behavior prohibitively costly for the culprit.

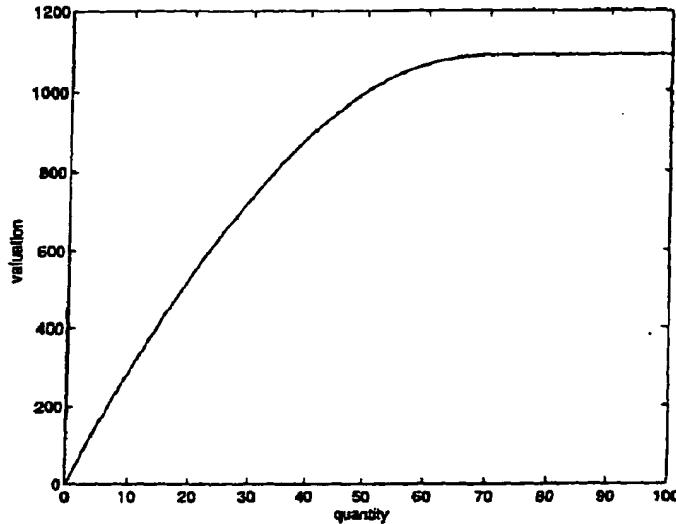


Figure 4: Parabolic valuation with  $\kappa = 0.5$  and  $\bar{q} = 70$

Another issue is how the convergence time scales with the number of bidders. We now consider this experimentally using the software described in Appendix B.

In all our simulations we let  $Q = 100$ . For each user, the valuation is strictly increasing and concave up to a maximum corresponding to a physical line capacity, and flat beyond that. Since, as shown by Proposition 3, only the second derivative of the valuation is needed to measure the efficiency of the PSP auction, a second order (parabolic) model is deemed sufficient. Thus we use valuations of the form:

$$\theta_i(z) = -\kappa_i(z \wedge \bar{q}_i)^2/2 + \kappa_i \bar{q}_i (z \wedge \bar{q}_i),$$

where  $\bar{q}_i$  is the line rate, and  $\kappa_i > 0$  (see Figure 4).

We generate our user population with independent random variables  $\{\theta_i(0)\}_I$  (corresponding to the maximum unit price the user would pay) uniformly distributed on  $[10, 20]$ , and  $\kappa_i = \theta_i(0)/\bar{q}_i$ , and  $\bar{q}_i$  uniformly distributed on  $[50, 100]$ . All players have a budget  $b_i = 100$ . The bid fee is fixed at  $\epsilon = 5$ . Each user has a bidding agent which can submit at most one bid per second (see Algorithm 1 in Appendix B).

With this set-up, the results are shown in Figures 5-6. Simulations were run for 11 population sizes ranging from 2 to 96 players. Each point is

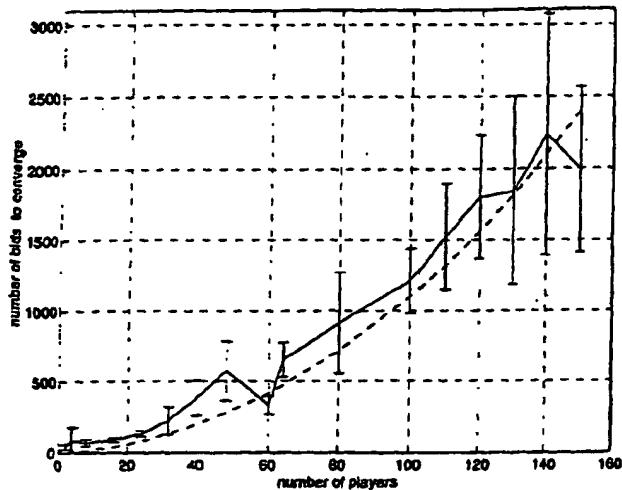


Figure 5: Mean (+/- std. dev.) number of bids – solid line. The dashed line is  $I + I^2/10$ .

simulated 10 times with new random valuations for all players. The overall mean is 11.9 bids per player. From Figure 5, the number of bids seems to grow as the square of the number of players.

The actual time to converge, shown in Figure 6, grows more slowly, since the computation of bids is done in parallel by all the players. In fact, for small numbers of players, the time decreases. This can best be explained as follows. Suppose there are only two players, with similar valuations. They will both start by asking for their maximum quantity, at their marginal valuation (which at their maximum quantity is near zero). Then as each sees the other's bid, each will reduce the quantity and increase the price a little bit. And they go on taking turns, gradually raising the market price until they reach an equilibrium. However if there are 10 players, in between two bids by the same player, the 9 others will already have bid up the price, so he will jump to higher price than if there was only one opponent. Thus the equilibrium market price will be reached more quickly. For large populations, this effect becomes small compared to the sheer volume of bids, and the convergence time starts to grow.

The trade-off between signaling and economic efficiency discussed in light of Proposition 3 is illustrated by Figures 7-8. Increasing the bid fee speeds

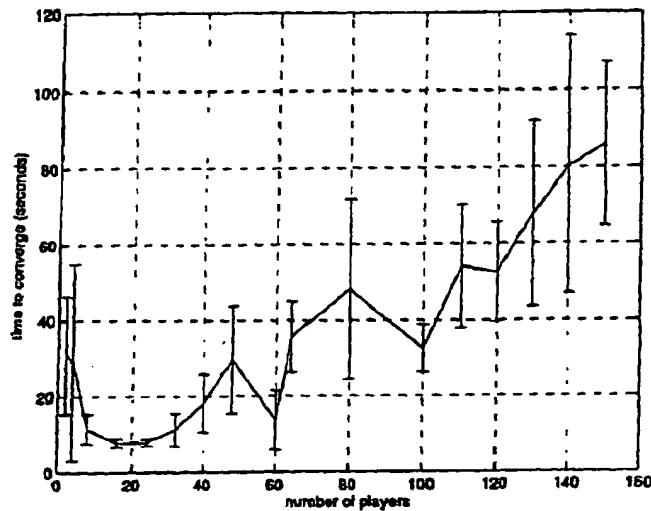


Figure 6: Mean (+/- std. dev.) convergence time in seconds (for a 1 second bid interval).

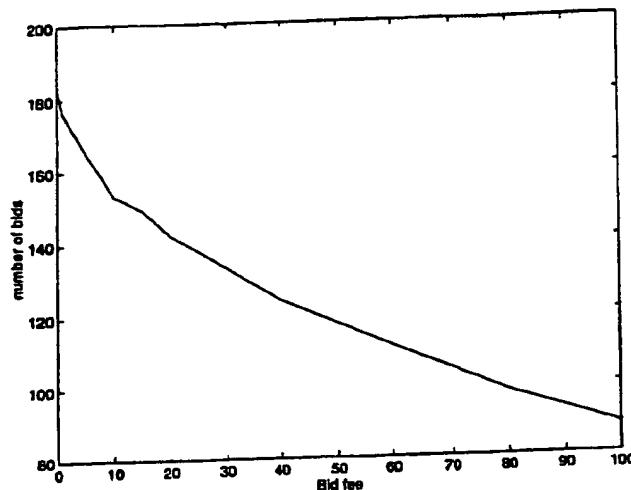
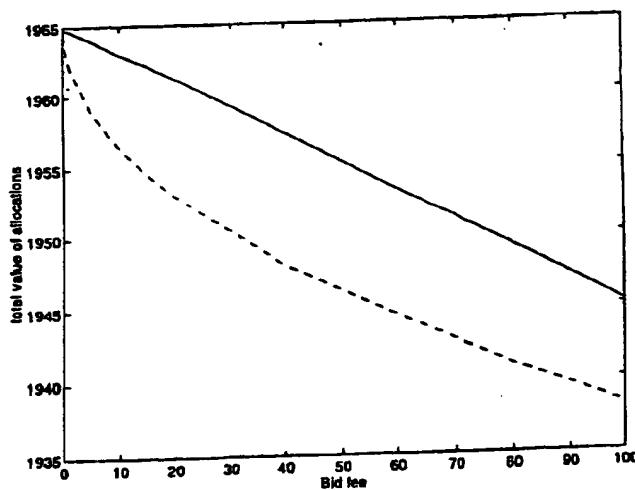
up convergence, at a cost of lost efficiency. A resource manager should select a bid fee which optimally balances the two for the particular context.

Figure 8 also illustrates the validity of the lower bound given by Proposition 3.

#### 4 Conclusion

Auctions are one of oldest surviving classes of economic institutions [...] As impressive as the historical longevity is the remarkable range of situations in which they are currently used. [22]

We proposed the progressive second price auction, a new auction which generalizes key properties of traditional single non-divisible object auctions to the case where an arbitrarily divisible resource is to be shared. We have shown that our auction rule, assuming an elastic-demand model of user preferences, constitutes a stable and efficient allocation and pricing mechanism in a network context. Even though we are motivated by problems of bandwidth and buffer space reservation in a communication network, the auction was formulated in a manner which is generic enough for use in a wide range

Figure 7: Number of bids to converge vs  $\epsilon$ Figure 8:  $\sum_i \theta_i(a_i^*)$  - solid line, and  $\max_a \sum_i \theta_i(a_i) - 4(\epsilon\kappa)^{-1/2}$  - dashed line, vs  $\epsilon$ .

of situations. In the sequel to this work, we show that the key results – namely incentive compatibility, equilibrium, and efficiency – generalize to a setting where multiple networked resources are auctioned, with users bidding on arbitrary but fixed routes and topologies [29, 31]. In related work, we consider the case of stochastically arriving players bidding for advance reservations (i.e. resources for a given period of time) [29, 30].

An interesting direction of future work is learning strategies, and evolutionary behavior which can emerge from repeated inter-action between the same players.

## 5 Acknowledgements

The authors would like to thank Wang Ke for his valuable help in the simulations.

## A Information-theoretic basis for the valuation

In general, valuations are simply assumed to be given as external factors. Indeed, the fundamental assumption in any market theory is that buyers know what the goods are worth to them. The “elastic demand” or “diminishing returns” nature of Assumption 1 is fully justified from a purely economic standpoint for virtually all resources in everyday life.

In the case of variable bandwidth, we can go even further by better quantifying what the goods are. For a user sending video, say, how much value is lost when the channel capacity goes from 1.5 to 1.2 Mbps? Ultimately, the value lies not in the amount of raw bandwidth but in the information that is successfully sent. Our goal in this section is to give a brief description of how Information Theory can be used for a bottom-up construction of bandwidth valuations – based on the fundamental thing the user cares about which is communication of information – and that such valuations will generally be of the type in Assumption 1.

Any information source has a function  $D(\cdot)$ , such that when compressed to a rate  $R$ , the signal has a distortion of at least  $D(R)$  [1]. The distortion is the least possible expected “distance” between the original and compressed signals, where the minimization is over all possible coding/decoding schemes. In this context, we make the distance measure the monetary cost of the error. This cost can be chosen, for example, to be proportional to some common measures like the mean squared error, the Hamming distance (probability of error), the maximum error, etc., or heuristic measures based on experiments with human perception. Given that modern source-coding techniques can, given a distortion measure, achieve distortions close to the theoretical lower bound[5], it is not unreasonable to use the rate-distortion curve as an indication of the value of the bandwidth share.

Let  $D_i(\cdot)$  be the distortion-rate function of  $\{X_i(t)\}$ , a stochastic process modeling the source of information associated with user  $i$ .  $X_i$  is encoded as  $Y_i$  which has a rate of  $R$  bits per second.

Shannon’s channel coding theorem[32] states that  $Y_i$  can be received without errors if and only if the channel has a capacity  $C > R$ . In our auction context, user  $i$  has capacity (bandwidth allocation)  $C = a_i$ , and thus has to suffer a distortion of at least  $D_i(a_i)$ . The value of the bandwidth is then

$$\theta_i(a_i) = \bar{\theta}_i - D_i(a_i), \quad (18)$$

where  $\bar{\theta}_i$  is the value of the full information.

The relevant properties of the distortion-rate functions are:

- when the rate is greater than the entropy of the source, the distortion is zero, and
- for many common source models and cost functions, the distortion-rate function is convex, and has a continuous derivative.

It is easy to see that, with these properties, (18) satisfies Assumption 1.

**Example 1:** Let  $\{X\}$  be a Bernoulli source, taking two values with probabilities  $p$  and  $1-p$ . Without loss of generality, let  $p \in [0, 1/2]$ . In this case, since the source is i.i.d., one can define the distortion on a per symbol basis. Using a Hamming cost function

$$d(X, Y) = 1_{\{X \neq Y\}},$$

i.e. assuming it costs one unit of money every time one bit is wrong, we have the distortion  $D = Ed(X, Y) = P(X \neq Y)$ . the rate-distortion function is

$$R(D) = [H(p) - H(D)]^+,$$

where  $H(x) = -x \log(x) - (1-x) \log(1-x)$ , and the distortion-rate function is the inverse function. It can be easily seen that  $D(R)$  is strictly convex and decreasing for  $0 \leq R < H(p)$ , and  $D(R) = 0$  for  $R \geq H(p)$ . The continuity of  $D'$  on  $0 \leq R < H(p)$  and  $R > H(p)$  is obvious. At the critical point ( $R = H(p)$ ,  $D = 0$ ), we have

$$\begin{aligned} \lim_{R \nearrow H(p)} D'(R) &= \lim_{D \searrow 0} 1/R'(D) \\ &= \lim_{D \searrow 0} 1/\log(D/1-D) \\ &= 0 \\ &= \lim_{R \searrow H(p)} D'(R). \end{aligned}$$

Thus continuity of  $D'$  holds throughout, and Assumption 1 is valid for the valuation of the form (18) for this source – see Figure 9.

**Example 2:** Let  $\{X\}$  be a Gaussian source with Markovian time-dependency, i.e. a covariance matrix  $\Phi = (\sigma^2 r^{|i-j|})_{i,j}$ ,  $r \in [0, 1]$ . Suppose we use the squared error cost, i.e. it costs one unit of money for one unit of energy in the error signal. Then, we have for low distortions  $D \leq (1-r)/(1+r)$ ,  $R(D) = \frac{1}{2} \log \frac{1-r^2}{D}$ , or

$$D(R) = (1 - r^2 \cdot \sigma^2 2^{-2R}),$$

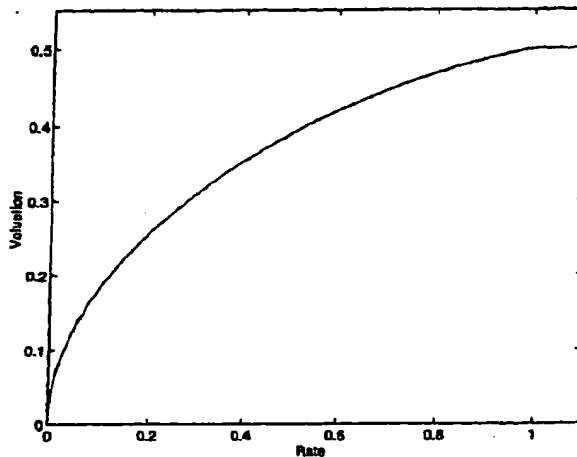


Figure 9: Distortion-rate based valuation for a Bernoulli  $p = 1/2$  source

and Assumption 1 clearly holds for (18). In the i.i.d. case ( $r = 0$ ), the formula holds for all  $R$ .

As the source models get more complex, it rapidly becomes impossible to give closed-form expressions for either  $R(D)$  or  $D(R)$ . Often parametric forms are available, and the functions can be evaluated numerically. Fortunately, the convexity property extends to a wide class of models, including for example auto-regressive sources, even when the generating sequence is non-Gaussian (see [1] for a full treatment of  $R(D)$ , including the above cases).

It can happen, e.g. for some video source models, that the R-D curve, which gives the best (R,D) pairs achievable by any coder/decoder, is not convex. But, for tractability, practical codecs are usually optimized on a convex hull of the space of possible (R,D) pairs[26, 27]. Thus, even when the theoretical D(R) curve is not convex, the actual distortion achieved in real-life systems almost always varies in a convex manner with the available bandwidth.

## B Simulation software and bidding algorithm

A prototype software agent based implementation of the auction game, called TREX, has been developed and extensively used since December 1995. Much of the intuition behind the mechanism design and the analysis in this work came from experiments done on this inter-active distributed auction game on the World Wide Web, using the Java programming language. The game can be played in real-time by any number of players from anywhere on the Internet [28].

Each user plays in the dynamic auction game using the following:

**Algorithm 1**    1 Let  $s_i = 0$ , and  $\hat{s}_{-i} = \emptyset$ . Start an independent thread which receives updates of  $\hat{s}_{-i}$ .

2 Compute the truthful  $\epsilon$ -best-reply of Proposition 1,  $t_i \in T_i \cap S_i^\epsilon(\hat{s}_{-i})$ .

3 If  $u_i(t_i; \hat{s}_{-i}) > u_i(s_i; \hat{s}_{-i}) + \epsilon$ , then send the bid  $s_i = t_i$ .

4 Sleep for 1 second.

5 Go to 2.

No assumption is made on the order of the turns. Players join the game at different times, and depending on the execution context of the client program, the sleep time of 1 second is more or less approximate. This, along with communication delays which make the times at which bids arrive at the server and updates at the clients essentially random times, make the distributed game completely asynchronous.

Algorithm 1 can be described as selfish and short-sighted. Selfish because it will submit a new bid if and only if it can improve its own utility (by more than the fee for the bid). Thus, the game can only converge<sup>7</sup> to an  $\epsilon$ -Nash equilibrium. And short-sighted because it does not take the extensive form of the game into account, i.e. does not use strategies which may result in a temporary loss but a better utility in the long run.

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<sup>7</sup>The fact that it does converge is established by Proposition 4 in Chapter 2 of [29].

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**APPENDIX B**

**Hold Option**

**Docket Number: 61624-04980**

## Chapter 4

### Spot and Derivative Markets in Admission Control

*Net game now risk, G. [87]*

In Chapters 2 and 3, we proposed a pure market approach for bandwidth pricing, where allocated capacities may vary during the lifetime of a flow, as players compete for resources through an auction game.

In this chapter, we propose a mechanism for circuit switched calls, wherein calls are admitted or rejected at (or soon after) their arrival time, and if admitted, get a fixed allocation of capacity, and have the option of securing the resource at a guaranteed maximum price for a guaranteed minimum duration. Thus the charge has two components:

- a market-based usage charge, where the user continuously pays the "instantaneous" market price (determined by second price auctions among recent arrivals); if the market price exceeds a user's bid price, that user is dropped unless
- upon arrival, the user pays a reservation fee (buys an option contract), which gives him the right to buy the capacity at any time in the future up to a

15 |

specified duration at his bid price. The user pays the market price as long as it is below his bid price. If at some point during the call, the market price exceeds the bid price, then the user automatically exercises the option, i.e., remains connected while paying no more than the bid price.

In other words, we introduce a "derivative" instrument to reduce the uncertainty inherent in the "spot" market mechanism. Naturally, this contract (or reservation) must itself be sold for a "fair" price. In the context of financial markets, the fair price is calculated with the Black-Scholes approach [8, 65], which is based on the idea that the option must be priced such that a perfectly hedged (i.e., riskless) combination of the derivative and the underlying equity will provide (locally in time) the same expected return as a risk-free security [36] (otherwise the derivative would present an arbitrage opportunity, i.e., unfair advantage, which would be exploited until its price rises).

In our context, the contract differs in that, rather than the right to buy once at a given strike price, it gives the right to buy repeatedly over a given duration, i.e., it is a series of "call options". Here the "fairness" we seek is that the reservation be priced in a way that reflects the probability that the system will become busier during the lifetime of the reservation, in order to avoid individually rational but socially sub-optimal behaviour. Specifically, we seek to avoid customers connecting at low periods and making a reservation limiting their usage price to an excessively low price for a very long time, to avoid rejoining when the prices are higher<sup>1</sup>.

A further difference from usual options is that, rather than the standard geometric Brownian motion model of e.g., a stock price, we have an underlying spot market process that is derived directly from our queueing system, via a heavy-traffic

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<sup>1</sup>A simplified form of this arbitrage is commonly observed with flat-rate priced dial-up Internet access, where some users remain logged on for very long periods of time even if they are not using the network, in order to avoid the chance of getting busy signals when they want to be on.

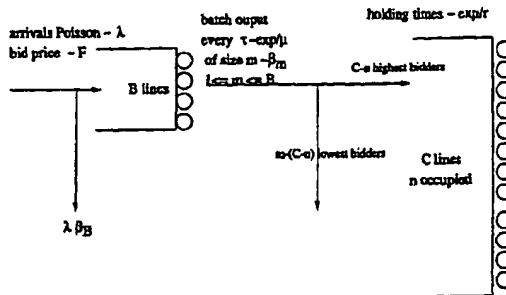


Figure 4-1: Queueing Model

approximation which leads to a diffusion process.

From the point of view of the user, this pricing mechanism is simply an initial reservation fee plus a per-minute (for example) usage charge. The novelty in the networking context is that both are market prices.

In Section 4.1., we derive a heavy-traffic diffusion approximation model for the queueing system. Using that, in Section 4.2., we derive the corresponding model for the spot market price. Then in Section 4.3., we introduce our formulation of the reservation as a derivative financial instrument. Finally, in Section 4.4. we present some simulation results using a real trace of traffic at a dial-up Internet access modem pool. For the sake of readability, detailed calculations are relegated to Sections 4.5. and 4.6.. Although not strictly necessary, a layman's grasp of financial markets is helpful in reading this chapter.

## 4.1. Queueing Model

### 4.1.1 Preliminaries

Customers arrive in a Poisson stream of rate  $\lambda$ , with i.i.d. bid prices distributed according to a distribution  $F$  (density  $f$ ). Call durations are exponentially dis-

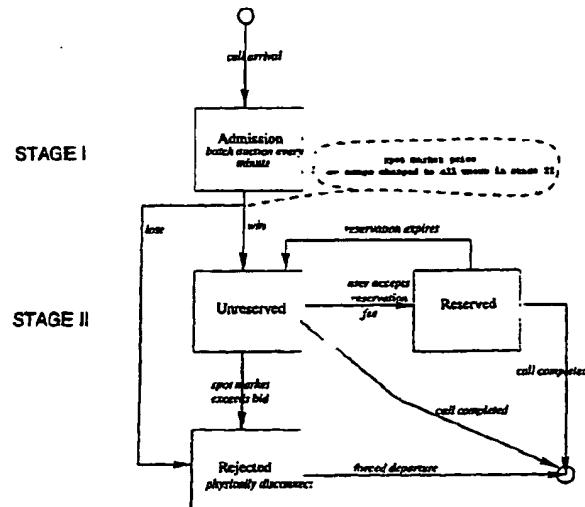


Figure 4-2: Pricing Mechanism

tributed with mean  $1/\tau$ . The queueing system is shown in Figure 4-1. It consists of two stages, with buffers of size  $B$  and  $C$  respectively.

The first stage consists of a second price auction, where the winners enter the second stage, and the losers leave the system. Specifically, the first stage has an exponentially distributed service time  $\tau$  with mean  $1/\mu$ . At a service completion instant, let  $m$  be the number of customers in the first stage and  $n$  the number in the second stage, at that time. The  $m$  first stage customers are ranked according to their bid prices. The  $(C-n)$  highest bids are accepted into the second stage, and the remaining  $m-(C-n)$  are dropped. The bid price of the highest dropped bid defines the new *spot market price*, which is valid until the next batch. If no customers are dropped, i.e.,  $m \leq C - n$ , then the spot market price is zero. Figure 4-2 shows the functioning of the mechanism.

The second stage is a  $C$  server queue with no waiting room, where each customer has an exponential service time (call duration) of mean  $1/\tau$ . The arrivals are according to a batch Poisson process with rate  $\mu$ , and batch sizes distributed

according to  $\{\beta_m\}_{m=0}^B$ . Let  $\rho = \lambda/\mu$ . For  $0 \leq m < B$ ,

$$\begin{aligned}\beta_m &\triangleq P\{m \text{ arrivals in } \tau\} \\ &= \frac{\rho^m}{(1+\rho)^{m+1}},\end{aligned}$$

and

$$\begin{aligned}\beta_B &= \sum_{m=B}^{\infty} \frac{\lambda^m \mu}{(\mu + \lambda)^{m+1}} \\ &= \left( \frac{\rho}{1+\rho} \right)^B\end{aligned}$$

In Section 4.1.2, the steady-state distribution of the occupancy of the second queue can be shown to be of product form. However, for the purpose of pricing the reservations, we need a (probabilistic) model of its *transient* behaviour in the future, given the current price. This is done in Section 4.1.3.

#### 4.1.2 Product-form solution

Let the state of the second queue (admitted calls) be  $X = (X_1, \dots, X_K)$ , where  $X_k = (X_{k,1}, \dots, X_{k,M_k})$ ,  $X_{k,i}$  is the bid price of the  $i$ -th player in the  $k$ -th batch,  $M_k$  is number of calls remaining from the  $k$ -th oldest batch, and  $K$  is the number of batches present. Let  $N = \sum_{k=1}^K M_k$ , the total number of active calls. The letters  $x, m_k$ , and  $n$  will denote generic values of  $X, M_k$  and  $N$ , respectively.

$X(t)$  is a continuous time Markov process. The transitions are of two types. On one hand we have departures of individual customers:

$$x = (x_1, \dots, x_k, \dots, x_K) \rightarrow x' = (x_1, \dots, e_i(x_k), \dots, x_K). \quad (4.1)$$

occurring with transition probability

$$\gamma(x, x') = r,$$

where  $e_i(x_k) = (x_{k,1}, \dots, x_{k,i-1}, x_{k,i+1}, \dots, x_{k,m_k})$ . On the other hand, we have batch arrivals:

$$x = (x_1, \dots, x_K) \rightarrow x' = (x_1, \dots, x_K, x_{K+1}), \quad (4.2)$$

occurring with probability intensity

$$\gamma(x, x') = \mu \beta_{m_{K+1}} \prod_{i=1}^{m_{K+1}} f(x_{K+1,i})$$

if  $0 \leq m_{K+1} < C - n$ , and

$$\gamma(x, x') = \mu \sum_{m=m_{K+1}}^B \beta_m F(\min_{1 \leq i \leq m_{K+1}} x_{K+1,i})^{m-m_{K+1}} \frac{m!}{m_{K+1}!(m-m_{K+1})!} \prod_{i=1}^{m_{K+1}} f(x_{K+1,i})$$

if  $m_{K+1} = C - n$ .

**Proposition 11** *The equilibrium probability of being in state  $x = (x_1, \dots, x_K)$  is*

$$\pi(x) = \pi(\emptyset) \left( \frac{\rho}{1+\rho} \right)^n \prod_{k=1}^K \frac{\mu}{r m_k} \prod_{i=1}^{m_k} f(x_{k,i}), \quad (4.3)$$

where the normalization constant is given by

$$\begin{aligned} 1/\pi(\emptyset) &= \sum_{\{x: 0 \leq n \leq C\}} \left[ \left( \frac{\rho}{1+\rho} \right)^n \left( \frac{\mu}{r} \right)^K / \prod_{k=1}^K m_k \right] \\ &= 1 + \sum_{n=1}^C \sum_{K=1}^n \sum_{m_1=1}^{B \wedge n} \sum_{m_2=1}^{B \wedge (n-m_1)} \dots \sum_{m_{K-1}=1}^{B \wedge (n-\sum_{i=1}^{K-2} m_i)} \frac{\left( \frac{\rho}{1+\rho} \right)^n \left( \frac{\mu}{r} \right)^K}{m_1 m_2 \dots m_{K-1} (n - \sum_{k=1}^{K-1} m_k)}. \end{aligned}$$

**Proof:** To prove the theorem, we first analyze and then conjecture the transition rates  $\gamma^*$  of the reversed process,  $X(-t)$ , and then verify that the following balance conditions:  
 $\forall x, x'$ ,

$$\pi(x)\gamma(x, x') = \pi(x')\gamma^*(x', x), \quad (4.4)$$

and  $\forall x$ ,

$$\sum_{x'} \gamma(x, x') = \sum_{x'} \gamma^*(x, x'), \quad (4.5)$$

hold. Indeed, if (4.4) and (4.5) are satisfied, then by Theorem 1.13 of [46],  $\pi$  is the equilibrium probability of the Markov process  $X(t)$ .

**Conjecture:**  $X(-t) = x$  has the following properties:

- new batches are formed by customers arriving according to a Poisson process of rate  $\lambda_0$ ;
- arrivals into existing batch  $k$  are according to a Poisson process of rate  $\lambda_k(x)$ . Arriving customers have a bid price i.i.d. with distribution  $F$ ;
- upon arrival into a batch, if the system is not full ( $n < C$ ), the customer is placed with equal probability in any position within the batch (from 1 to  $m_k + 1$ ). If the system is full, the new customer departs immediately;
- batch  $k$  has an exponentially distributed service time, with mean  $1/\mu_k(x)$ .

The rates for the reverse process are the following. First,

$$\lambda_0 = \mu(1 - \beta_0).$$

Second, if  $n < C$  or  $k < K$ ,

$$\mu_{n+1,k} = \beta_{m_k} \left( \frac{1+\rho}{\rho} \right)^{m_k} \tau m_k,$$

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and

$$\lambda_k(x) = \begin{cases} \mu_k(x)\rho & \text{if } 1 \leq m_k < B \\ 0 & \text{if } m_k = B \end{cases}$$

Third, when  $n = C$ , and  $k = K$ , i.e., for the last batch when the system is full,

$$\mu_K(x) = \sum_{m=m_K}^B \beta_m F(\min_{1 \leq i \leq m_K} x_{k,i})^{m-m_K} \frac{m!}{m_k!(m-m_k)!} \left(\frac{1+\rho}{\rho}\right)^{m_k} r m_k,$$

and

$$\lambda_K(x) = r m_K - \mu_K(x). \quad (4.6)$$

Note that in this last case all the arrivals immediately depart.

We begin by verifying (4.4). For the transitions of type (4.1), i.e., corresponding to departures in the forward process, consider a departure from the  $k$ -th batch. When  $m_k > 1$ , (4.3) yields

$$\pi(x_1, \dots, x_k, \dots, x_K)r = \pi(x_1, \dots, e_i(x_k), \dots, x_K) \frac{\rho}{1+\rho} \frac{m_k-1}{m_k} f(x_{k,i})r.$$

Since  $m_k - 1 \leq B - 1$ , we have  $\beta_{m_k-1} = \rho^{m_k-1}/(1+\rho)^{m_k}$ , and therefore  $\mu_k(x') = r \cdot m_k - 1)/(1-\rho)$ . On the right hand side the system is not full ( $n' = n - 1 \leq C - 1$ ). Hence  $\lambda_k(x') = \mu_k(x')\rho = \frac{\rho}{1+\rho} r(m_k - 1)$ . Thus we have

$$\pi(x)r = \pi(x') \frac{\lambda_k(x')}{m_k} f(x_{k,i}).$$

Now, since arrivals in the reverse process take any position in their batch with equal probability,  $\lambda_k(x')/m_k$  is precisely the arrival rate into position  $i$  in batch  $k$ , i.e.,  $\lambda_k(x')/m_k = \gamma^*(x', z)$ , as desired. If  $m_k = 1$ , (4.3) yields

$$\begin{aligned} \pi(x_1, \dots, (x_{k,1}), \dots, x_K)r &= \pi(x_1, \dots, x_{k-1}, x_{k+1}, \dots, x_K) \frac{\rho}{1+\rho} \frac{\mu}{r} f(x_{k,1})r \\ &= \pi(x_1, \dots, x_{k-1}, x_{k+1}, \dots, x_K) \lambda_0 f(x_{k,1}), \end{aligned}$$

as desired.

For the transitions of type (4.2), if  $1 \leq m_{K+1} < C - \sum_{k=1}^{K-1} m_k$ ,

$$\begin{aligned}\pi(x)\beta_{m_{K+1}}\mu \prod_{i=1}^{m_{K+1}} f(x_{K+1,i}) &= \pi(x')m_{K+1} \frac{r}{\mu} \left(\frac{1+\rho}{\rho}\right)^{m_{K+1}} \beta_{m_{K+1}}\mu \\ &= \pi(x')\mu_{K+1}(x'),\end{aligned}$$

as desired. If  $m_{K+1} = C - \sum_{k=1}^{K-1} m_k$ ,

$$\begin{aligned}\pi(x)\mu \sum_{m=m_{K+1}}^B \beta_m F(\min_{1 \leq i \leq m_{K+1}} x_{K+1,i})^{m-m_K} \frac{m!}{m_{K+1}!(m-m_{K+1})!} \prod_{i=1}^{m_{K+1}} f(x_{K+1,i}) \\ &= \pi(x')m_{K+1} \frac{r}{\mu} \left(\frac{1+\rho}{\rho}\right)^{m_{K+1}} \mu \sum_{m=m_{K+1}}^B \beta_m F(\min_{1 \leq i \leq m_{K+1}} x_{K+1,i})^{m-m_K} \frac{m!}{m_{K+1}!(m-m_{K+1})!} \\ &= \pi(x')\mu_{K+1}(x'),\end{aligned}$$

as desired.

We now verify (4.5). The left-hand side is the total rate at which the forward process  $X(t)$  leaves state  $x$ , which is  $\mu(1-\beta_0)+nr$ . The right-hand side is the total rate of leaving state  $x$  in the reverse process. Consider first a state where the system is not full ( $n < C$ ).

The total rate is then (letting  $k$  range from 1 to  $K$ )

$$\begin{aligned}\lambda_0 + \sum_k \lambda_k(x) + \mu_k(x) &= \mu(1-\beta_0) + (1+\rho) \sum_{m_k < B} \mu_k(x) + \sum_{m_k=B} \mu_k(x) \\ &= \mu(1-\beta_0) + (1+\rho) \sum_{m_k < B} \frac{\rho^{m_k}}{(1+\rho)^{m_k+1}} \left(\frac{1+\rho}{\rho}\right)^{m_k} rm_k \\ &\quad + \sum_{m_k=B} \left(\frac{\rho}{1+\rho}\right)^{m_k} \left(\frac{1+\rho}{\rho}\right)^{m_k} rm_k \\ &= \mu(1-\beta_0) + \sum_k rm_k = \mu(1-\beta_0) + rn,\end{aligned}$$

as desired. Now when  $n = C$ , the rate is

$$\begin{aligned} & \lambda_0 + \sum_{k=1}^{K-1} (\mu_k(x) + \lambda_k(x)) + \lambda_K(x) + \mu_K(x) \\ &= \mu(1 - \beta_0) + (n - m_K)r + rm_K - \mu_K(x) + \mu_K(x), \end{aligned}$$

where the second term is derived as in the  $n < C$  case above, and the third term comes from the definition (4.6). Then the right hand side is  $\mu(1 - \beta_0) + nr$ , as desired.  $\square$

#### 4.1.3 Diffusion approximation

We develop a tractable approximation for the queue occupancy process  $N_t$ , by scaling the system following the well-known heavy traffic approach [37, 9, 97, 98, 18], applied here to a system with batch arrivals.

Let  $\alpha \triangleq \frac{\mu m}{r}$ ,  $\bar{m} \triangleq \mathbb{E}M = \sum_{m=0}^B m\beta_m$ , and  $v^2 \triangleq \mathbb{E}(M^2) = \sum_{m=0}^B m^2\beta_m$ . Consider a sequence of scaled systems, indexed by  $l = 1, 2, \dots$ , with capacity  $C^{(l)} = l\alpha + \gamma\sqrt{l\alpha}$ , where  $\gamma$  is arbitrary, and batch arrival rate  $\mu^{(l)} = l\mu$ . Let

$$Z_t^{(l)} \triangleq \frac{N_t^{(l)} - l\alpha}{\sqrt{l\alpha}}, \quad (4.7)$$

the translated and scaled version of the queue occupancy of the  $l$ -th system. The drift and diffusion coefficient respectively are

$$\begin{aligned} a^{(l)}(z) &\triangleq \lim_{\delta \rightarrow 0} \mathbb{E} \left[ \frac{Z_{t+\delta}^{(l)} - Z_t^{(l)}}{\delta} \mid Z_t^{(l)} = z \right] \\ &= \begin{cases} -rz & \text{if } z < \gamma \\ -r(\gamma + \sqrt{l\alpha}) & \text{if } z \geq \gamma. \end{cases} \quad (4.8) \end{aligned}$$

and

$$\begin{aligned}\sigma^{(l)2}(z) &\triangleq \lim_{\delta \rightarrow 0} \mathbb{E} \left[ \frac{(Z_{t+\delta}^{(l)} - Z_t^{(l)})^2}{\delta} | Z_t^{(l)} = z \right] \\ &= \begin{cases} r(1 + v^2/\bar{m} + z/\sqrt{l\alpha}) & \text{if } z < \gamma \\ r(1 + \gamma/\sqrt{l\alpha}) & \text{if } z \geq \gamma. \end{cases} \quad (4.9)\end{aligned}$$

The basic idea is that as  $l \rightarrow \infty$ ,  $Z_t^{(l)} \Rightarrow Z_t$ , (where  $\Rightarrow$  denotes weak convergence, i.e., the probability distributions converge), where  $Z_t$  is the diffusion process which solves the stochastic differential equation

$$dZ_t = a(Z_t) dt + \sigma(Z_t) dW_t, \quad (4.10)$$

where  $W_t$  is a Brownian motion.

Note that, for  $\gamma < \infty$ , the offered load  $\alpha^{(l)}/C^{(l)} = (1 + \gamma/\sqrt{l\alpha})^{-1} \nearrow 1$ , which makes it a "heavy traffic" approximation.

The coefficients  $a(\cdot)$  and  $\sigma^2(\cdot)$  are obtained by letting  $l \rightarrow \infty$  in (4.8) and (4.9) respectively. In the original approach of [37],  $Z_t^{(l)} \Rightarrow Z_t$  holds if  $a(\cdot)$  and  $\sigma^2(\cdot)$  are continuous, which is clearly not the case here, since both have jumps at  $z = \gamma$ . However, based on a theorem of Borovkov [9], Whitt [98] proposes a conditioning heuristic to derive approximate steady-state blocking probabilities for the  $G/GI/s/0$  (i.e.,  $s$  servers, no waiting room) queue from the diffusion approximation for the corresponding  $G/GI/\infty$  system. Our approach here will be to apply the heuristic to the diffusion process itself<sup>2</sup>, conditioning on  $Z_t \leq \gamma$ .

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<sup>2</sup>[98] p. 694 states that, for exponential service times, the heuristic is also applicable to the diffusion process itself.

## 4.2. Diffusion Models of the Market Prices

### 4.2.1 Spot Price

Suppose that at time  $t$ , the queue occupancy is  $N_t = n$ , and a batch of size  $m$  arrives. The admission decision is made, and a new spot price results. In this section, we want to derive a model which will tell us in some sense the future evolution of this price.

Consider the mechanism by which the price arises. Of the  $m$  new customers, those with the  $C - n$  highest bid prices will be admitted, and the spot price will be the  $(C - n + 1)$ -th highest bid price. Recall that the bid prices are i.i.d., with distribution  $F$ , which we assume to be smooth. The probability that the price is  $x$  is the probability that in  $m$  draws, one will equal  $x$ , and of the remaining  $m - 1$ ,  $C - n$  will be greater than  $x$ , and  $(m - 1) - (C - n)$  will be less than or equal to  $x$ , which is

$$\begin{aligned} f_{n,m}(x) &\triangleq m f(x) \frac{(m-1)!}{(C-n)![((m-1)-(C-n))!]} [F(x)]^{m-C+n-1} [1-F(x)]^{C-n} \\ &= \frac{m!}{(C-n)!(m-1-C+n)!} [F(x)]^{m-C+n-1} [1-F(x)]^{C-n} f(x). \end{aligned}$$

The distribution is

$$\begin{aligned} F_{n,m}(x) &\triangleq \int_0^x f_{n,m}(y) dy \\ &= \int_0^{F(x)} \frac{m!}{(C-n)!(m-1-C+n)!} u^{m-C+n-1} (1-u)^{C-n} du, \end{aligned}$$

where the last equality comes from substituting the previous expression, and making the change of variables  $u = F(y)$ .  $du = f(y) dy$ .

Let us consider a "first-order" approximation, the expected spot price given the

occupancy  $n$  and the size of the batch arrival  $m$ :

$$\begin{aligned}\psi_{n,m} &\triangleq \int_0^\infty [1 - F_{n,m}(x)] dx \\ &= \int_0^\infty \int_{F(x)}^1 g(u, n, m) du dx \\ &= \int_0^1 F^{-1}(u) g(u, n, m) du,\end{aligned}$$

where

$$g(u, n, m) \triangleq \frac{m!}{(C-n)!(m-1-C+n)!} u^{m-C+n-1} (1-u)^{C-n}. \quad (4.11)$$

#### 4.2.2 Diffusion model of the spot price

For the purpose of pricing a reservation beginning at time  $t$ , we would like to characterize the future evolution of the market price, in terms of what is known at or just before  $t$ , namely the occupancy  $N_t$ . Thus, it is natural to consider the process

$$\psi(N_t) = \mathbb{E}[\psi_{N_t, m}] = \sum_{m=0}^B \beta_m \psi_{N_t, m}.$$

**Remark:** We prefer the above expression to one that uses the information in the batch that arrives at time  $t$ , since computing the *option* prices based on the new arrivals would add to the delay in the admission decision. By not waiting for the new information, we can use the time in between batches for computation of option prices. Of course the *spot* price results from the auction among the new bids, but that is a very simple computation.

Now, we approximate the price by a sequence of functions of the centered and scaled process  $Z_t^{(l)}$ ,

$$P^{(l)}(z) \triangleq \psi(z\sqrt{\alpha l} + \alpha l).$$

Then, Ito's rule (see, e.g., [45]) along with (4.10) yields the stochastic differential equation for  $P$ :

$$dP_t^{(l)} = \left[ \frac{dP^{(l)}}{dz} a^{(l)} + \frac{1}{2} \frac{d^2 P^{(l)}}{dz^2} \sigma^{(l)2} \right] dt + \frac{dP^{(l)}}{dz} \sigma^{(l)} dW_t. \quad (4.12)$$

Since

$$P^{(l)}(z) = \sum_{m=0}^B \beta_m \int_0^1 F^{-1}(u) g^{(l)}(u, \sqrt{\alpha l} z + \alpha l, m) du,$$

where  $g^{(l)}$  is defined as  $g$  with  $C^{(l)}$  instead of  $C$ , it follows that

$$\frac{dP^{(l)}}{dz}(z) = \sum_{m=0}^B \beta_m \int_0^1 F^{-1}(u) \frac{\partial}{\partial z} g^{(l)}(u, \sqrt{\alpha l} z + \alpha l, m) du, \quad (4.13)$$

and

$$\frac{d^2 P^{(l)}}{dz^2}(z) = \sum_{m=0}^B \beta_m \int_0^1 F^{-1}(u) \frac{\partial^2}{\partial z^2} g^{(l)}(u, \sqrt{\alpha l} z + \alpha l, m) du. \quad (4.14)$$

Now (4.12), along with (4.8), (4.9), (4.13) and (4.14), constitutes a diffusion model for the spot market price  $P_t$ , as  $l \rightarrow \infty$ . From the results derived in Sections 4.5. and 4.6., we can approximate (4.12) by  $dP_t = 0$  when  $z < \gamma$ , and

$$\begin{aligned} dP_t = & \sum_{m=4V^2}^B \beta_m \left[ rC \left( -\frac{1}{m} P_t + \frac{m-1}{m} K_1^{(m)} + \frac{1}{2} K_2^{(m)} \right) dt \right. \\ & \left. + \sqrt{rC} \left( \frac{1}{m} P_t - K_1^{(m)} \right) dW_t \right], \end{aligned} \quad (4.15)$$

when  $z = \gamma$ .  $V$  is a constant related to the accuracy to which we want to evaluate  $g$ , and can typically be taken to be 1 (see Section 4.6.).  $\forall m$ , the constants  $K_1^{(m)}$ ,  $K_2^{(m)}$ , which depend only on the bid price distribution  $F$ , are defined (and can be computed) by (4.23) and (4.25) respectively.

**Remark:**  $z = \gamma$  corresponds to  $N_t = C$ , i.e., the system is full. Thus we have a model where the spot price remains constant when the system is below capacity,

and varies when the system is full. This corresponds to what one would intuitively expect from the auction mechanism (with the constant price 0 when the system is not full). Note that we do not attempt to model the transitions between the two regimes, so this model is meaningful only during periods when the system is almost always full.

#### 4.3. Computing Reservation Fees: the Derivative Market

Taking the current market price  $P_t$  as initial condition, the solution of (4.15) provides a stochastic model of the future prices  $\{P_\tau\}_{\tau \geq t}$  (this solution is unique in distribution – see [45]). From that, we now determine the price of a reservation, arriving at time  $t$ , with a holding time  $T$ , and a bid price  $p$ . The value, or “fair price”, at time  $t$  of an option to buy a security for a “strike price”  $p$  at a specific future date<sup>3</sup>  $\tau \geq t$  is (see [45, 36])

$$\zeta_\tau \triangleq \mathbb{E}(P_\tau - p)^+.$$

In our context, this concept must be extended in the following straightforward manner: we define the reservation as a *hold option*, a new kind of derivative instrument which is an option to buy repeatedly at every time instant from  $t$  to  $t + T$ . Thus the reservation fee should be

$$\phi(t, T, p) \triangleq \int_t^{t+T} \zeta_\tau d\tau. \quad (4.16)$$

If  $K_1 = K_2 = 0$ ,  $\zeta_\tau$  is given explicitly by the Black-Scholes formula [36]. For the

<sup>3</sup>This is called a European option, as opposed to an American option, which is the right to buy at any time between  $t$  and  $\tau$ .

Note that we take the risk-free interest rate to be 0.

more general form (4.15), which we re-write as

$$dP_\tau = (AP_\tau + K_3) dt + (DP_\tau + K_4) dW_t, \quad (4.17)$$

for  $\tau \geq t$ , with known initial condition  $P_t$ , the solution is (see [45], Section 5.6)

$$P_\tau = S_\tau \left[ P_t + \int_t^\tau \frac{K_3 - DK_4}{S_u} du + \int_t^\tau \frac{K_4}{S_u} dW_u \right], \quad (4.18)$$

where

$$S_\tau \triangleq \exp \left[ (A - D^2/2)(\tau - t) + \int_t^\tau D dW_u \right].$$

Knowing the set of distributions  $\mathcal{P}_t \triangleq \{\mathbb{P}(P_\tau \leq x|P_t), \forall x : \tau > t\}$ , we can evaluate  $\zeta_\tau$ , for  $\tau > t$  and the reservation fee  $\phi$  follows.

#### 4.4. Simulations

In this section, we simulate the pricing mechanisms presented in the preceding sections, applied to dial-up Internet access.

Our data set consists of all the connections that were made to the Columbia University modem pool between April 4th and May 10th 1998. At that time, the system, with a total of 316 lines, was very heavily loaded (except for the period roughly from 2AM to 7AM every day, essentially all lines are always busy). There was a total 669,994 calls, from 9451 different users, with an average inter-arrival time of 5.59 seconds, and an average call duration of 1299.3 seconds (see Figure 4-3).

The unsatisfied demand, i.e., the calls which encounter busy signals, are not logged in our data trace, since the blocking occurs in the public telephone network. Our data trace only includes admitted calls. Thus feeding the trace into a system of 316 lines would result in no blocking, which would not be interesting since blocked

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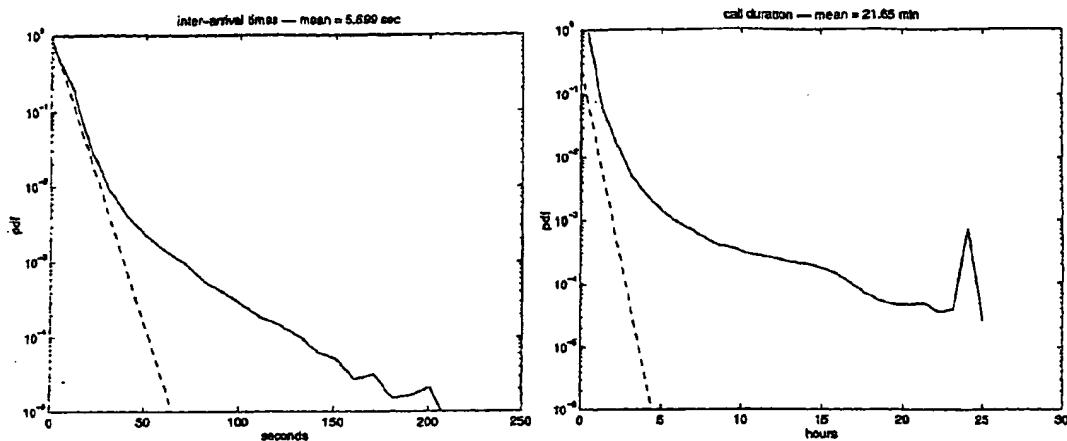


Figure 4-3: Modem pool call statistics (solid) and equivalent – same mean – exponential distributions (dashed)

calls are what gives rise to the market price. By scaling down the capacity of system in our simulation, we can recreate the congestion. The traffic was fed into a system running with 202 lines, of which 18 were allocated to the first stage, and 184 to the second stage. The call arrival times and bid durations are from the real traffic trace (however the simulation was run at 10× real-time speed), while bid prices are randomly generated from  $F$ , the uniform distribution on  $[0, 1]$ .

Rather than explicitly evaluating the set of distributions functions  $\mathcal{P}_t$  resulting from (4.18), our technique consists of Monte-Carlo simulation. Specifically, at each time when a reservation has to be computed, beginning at the known current price  $P_t$ , we generate sample paths by the difference equation corresponding to (4.15), which results in a set of histograms (one for each value of  $\tau$ ) corresponding to  $\mathcal{P}_t$ . Figure 4-4 shows a representative example.

Since the arrival process is clearly time-varying, the implemented reservation mechanism makes real-time estimates of  $\lambda$ , and  $r$ , to evaluate the constants  $A$ ,  $K_3$ ,  $D$  and  $K_4$ , using the formulas in Section 4.6..

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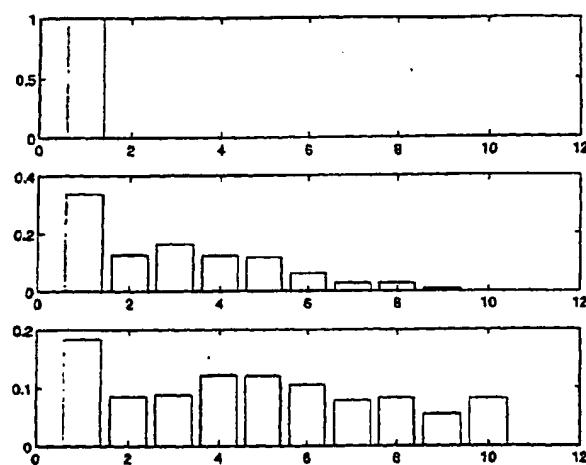


Figure 4-4: Histograms of  $P_\tau$ , for  $P_0 = 0$  and  $\tau = 0, 1, 2$  minutes; the bins represent price levels  $[0, 0.1), [0.1, 0.2), \dots, [0.9, 1)$ .

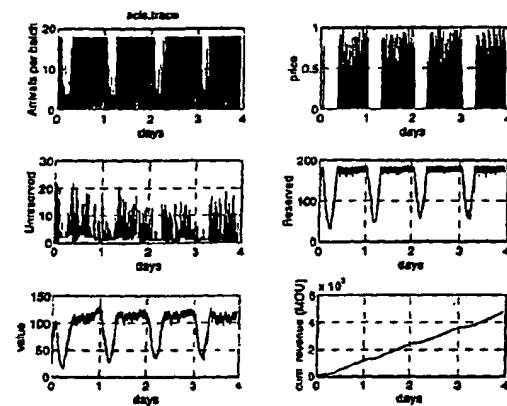


Figure 4-5: Simulation trace

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Figure 4-5 shows a snapshot of the simulation trace. The first plot shows the number of users arriving in each batch interval, where  $\frac{1}{\mu} = 1$  minute. The second plot shows the spot price process resulting from the batch auctions. As expected, prices are zero when the system is lightly loaded during the night. The average price including days and nights is 0.2055.

The third and fourth plots show the number of users in the second stage, in the reserved and unreserved states. In this simulation, all users accept the reservation offer, thus they only remain in the unreserved state for a brief period, from the time the offer is sent by the auctioneer (which is a server program), until the acceptance returns from the user (which is a client program on a different host).

In the fifth plot, "value" is the sum of the bid prices in the second stage (reserved+unreserved): this represents the social welfare, or the total value that the users are getting from the system. Without pricing, i.e., if all the users were simply admitted on a FCFS basis, the average value during peak periods would be  $184 \times 0.5 = 92$ , since the average bid price is 0.5. Here the average value during the day is approximately 110, i.e., the pricing mechanism yields a 20% gain in efficiency. In other words, without pricing there are many times when a user who values access less is denying a user who values it more. The efficiency is shown more clearly in Figure 4-6, normalized so that the horizontal line corresponds to the FCFS system. Note that there is an efficiency gain of up to 10% even during off-peak hours, because some of the off-peak users arrived during peak hours.

The sixth plot of Figure 4-5 shows the cumulative revenue from usage charges and reservation fees. Since the batch interval is one minute, and usage charges are assessed on a minutely basis, we take the price of 1 as the "full price" reference level, i.e., one unit of revenue is when 1 line is charged a price of 1 for 1 minute. The unit is called an MOU (minute-of-use). As can be seen on the plot, the revenue for

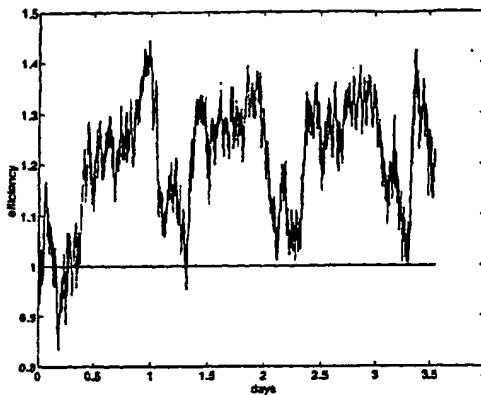


Figure 4-6: Efficiency: 1.0 = first-come first-served system

one day is approximately 125,000 MOUs. The 184 lines multiplied by 1440 minutes in a day yield about 264,960 potential minutes of use. Thus, on average, a modem line generates 0.47 MOUs of revenue per minute, including usage and reservation charges.

Figures 4-7 and 4-8 show how the reservation fee offsets any attempt to "arbitrage". Durations range from 0 to 24 hours, and the reservation fees range from 0 to 640 MOUs, with the highest ratio being 1 MOU of reservation fee per minute of reservation duration, as can be seen by the straight line upper-bounding the scatter plot in Figure 4-7. The highest ratio (most expensive reservation) would be for a user who gets into the system with a bid price near zero and requests a long reservation at a time when the arrival rate is high. That is illustrated by Figure 4-8. A user asking for a long duration reservation at a low bid price must pay a higher fee which compensates for the expected future rise in prices. The vertical variations are due to the market price at the moment the reservation offer is made, where the higher end are reservations made when the market price is high.

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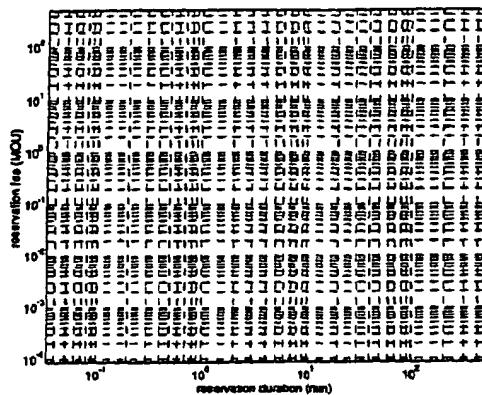


Figure 4-7: The reservation fee is at most  $1 \times$  duration.

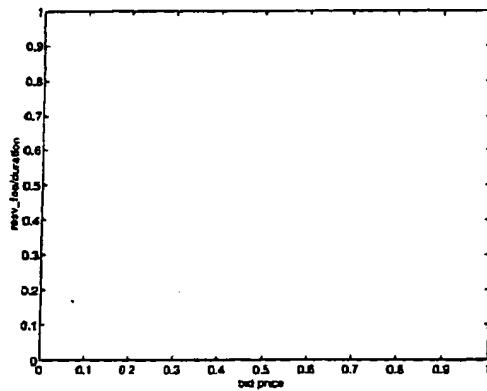


Figure 4-8: Reservations are proportionally more expensive for low bid prices.

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#### 4.5. Approximations related to $g$

We first seek a convenient approximation of (4.11). Fix  $V > 0$ .

##### 4.5.1 Case $mu(1-u) > V^2$

By the Central Limit Theorem, the binomial density tends to the normal density, i.e., for  $0 < u < 1$ , we have

$$\frac{m!}{k!(m-k)!} u^{m-k} (1-u)^k \approx \frac{1}{\sqrt{2\pi mu(1-u)}} \exp \left\{ \frac{-[k - (1-u)m]^2}{2u(1-u)m} \right\}.$$

Thus,

$$\begin{aligned} g(u, n, m) &= \frac{m!}{(C-n)!(m-1-C+n)!} u^{m-C+n-1} (1-u)^{C-n} \\ &\approx \frac{m-C+n}{u} \frac{1}{\sqrt{2\pi mu(1-u)}} \exp \left\{ \frac{-[C-n-(1-u)m]^2}{2u(1-u)m} \right\}. \end{aligned}$$

Differentiation yields

$$\frac{\partial g}{\partial n}(u, n, m) \approx g(u, n, m) \left[ \frac{1}{m-C+n} + \frac{C-n-m(1-u)}{u(1-u)m} \right]$$

and

$$\begin{aligned} \frac{\partial^2 g}{\partial n^2}(u, n, m) &\approx g(u, n, m) \left\{ \left[ \frac{1}{m-(C-n)} + \frac{C-n-m(1-u)}{u(1-u)m} \right]^2 \right. \\ &\quad \left. - \left( \frac{1}{m-C+n} \right)^2 - \frac{1}{u(1-u)m} \right\}. \end{aligned}$$

Now, recalling that  $C^{(l)} = l\alpha + \gamma\sqrt{l\alpha}$ , we get

$$g^{(l)}(u, \sqrt{\alpha}z + \alpha l, m) \approx \frac{m - (\gamma - z)\sqrt{\alpha l}}{u} \times$$

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$$\frac{1}{\sqrt{2\pi mu(1-u)}} \exp \left\{ \frac{-[(\gamma - z)\sqrt{\alpha l} - (1-u)m]^2}{2u(1-u)m} \right\}.$$

When  $z \neq \gamma$ , as  $l \rightarrow \infty$  the exponential goes to zero faster than the other factors grow, thus  $\frac{\partial g}{\partial u}(u, \sqrt{\alpha l}z + \alpha l, m), \frac{\partial^2 g}{\partial u^2}(u, \sqrt{\alpha l}z + \alpha l, m) \rightarrow 0$ . Therefore, by (4.13) and (4.14),  $\frac{dP^{(l)}}{du}(z) = \frac{d^2 P^{(l)}}{du^2}(z) \rightarrow 0$ .

But when  $z = \gamma$ ,

$$g^{(l)}(u, \sqrt{\alpha l}z + \alpha l, m) \sim \frac{m}{u} \frac{1}{\sqrt{2\pi mu(1-u)}} \exp \left\{ \frac{-(1-u)m}{2u} \right\} \quad (4.19)$$

$$\frac{\partial}{\partial z} g(u, \sqrt{\alpha l}z + \alpha l, m) \sim \sqrt{\alpha l} \left( \frac{1}{m} - \frac{1}{u} \right) g^{(l)}(u, \sqrt{\alpha l}z + \alpha l, m) \quad (4.20)$$

$$\frac{\partial^2}{\partial z^2} g(u, \sqrt{\alpha l}z + \alpha l, m) \sim \alpha l \left[ \left( \frac{1}{m} - \frac{1}{u} \right)^2 - \frac{1}{m^2} - \frac{1}{u(1-u)m} \right] \times g^{(l)}(u, \sqrt{\alpha l}z + \alpha l, m) \quad (4.21)$$

as  $l \rightarrow \infty$ , where  $\sim$  indicates that it is an approximation valid for large  $m$ .

#### 4.5.2 Case $mu(1-u) \leq V^2$

Here we consider the neighborhoods of  $u = 1$  and  $u = 0$ , where the normal approximation is no longer valid. Indeed

$$g(1, n, m) = m 1_{\{C\}}(n),$$

and

$$g(0, n, m) = m 1_{\{C-m+1\}}(n),$$

where  $1_S(x) = 1$  if  $x \in S$  and = 0, otherwise.

Thus, for  $u$  near 1, we write

$$\begin{aligned} g^{(l)}(u, \sqrt{\alpha l}z + \alpha l, m) &\approx m1_{\left(\gamma - \frac{1}{2\sqrt{\alpha l}}, \gamma + \frac{1}{2\sqrt{\alpha l}}\right]}(z) \rightarrow m1_{\{r\}}(z) \\ \frac{\partial}{\partial z}g^{(l)}(u, \sqrt{\alpha l}z + \alpha l, m) &\approx m \left[ \delta(z - \gamma + \frac{1}{2\sqrt{\alpha l}}) - \delta(z - \gamma - \frac{1}{2\sqrt{\alpha l}}) \right] \rightarrow 0 \\ \frac{\partial^2}{\partial z^2}g^{(l)}(u, \sqrt{\alpha l}z + \alpha l, m) &\approx m \left[ \delta'(z - \gamma + \frac{1}{2\sqrt{\alpha l}}) - \delta'(z - \gamma - \frac{1}{2\sqrt{\alpha l}}) \right] \rightarrow 0, \end{aligned}$$

where  $\delta$  is the Dirac- $\delta$  function.

For  $u$  near 0, we get the same approximation with  $\gamma$  replaced by  $\gamma - \frac{m-1}{\sqrt{\alpha l}}$ , which becomes identical to the above as  $l \rightarrow \infty$ .

Here  $\forall z$ ,  $\frac{dP^{(l)}}{dz}(z) = \frac{d^2P^{(l)}}{dz^2}(z) \rightarrow 0$ .

#### 4.6. Approximating the Drift and Diffusion Coefficients

When  $z \neq \gamma$ , by the results of 4.5.1 and 4.5.2, the right-hand side of both (4.13) and (4.14)  $\rightarrow 0$ , as  $l \rightarrow \infty$ . Therefore, by the finiteness of (4.8) and (4.9), the right-hand side of (4.12)  $\rightarrow 0$ , as  $l \rightarrow \infty$ .

Thus we need only be concerned with the regime  $z = \gamma$ . Let

$$u_{V,m} \triangleq \frac{1 - \sqrt{1 - 4V^2/m}}{2},$$

which satisfies  $u_{V,m}(1 - u_{V,m})m = V^2$ .  $V$  can be interpreted as the smallest variance at which the binomial distribution is sufficiently close to the normal distribution. Typically  $V$  can be taken to be about 1.

Now we will approximate  $g$  and its partials in three regions:  $[0, u_{V,m}]$ ,  $[u_{V,m}, 1 - u_{V,m}]$ , and  $(1 - u_{V,m}, 1]$ .

In the first and last regions, as shown in Section 4.5.2, the coefficients tend

to 0. Thus we need only focus on the middle region, where the approximations of Section 4.5.1 apply. Also, if  $m < 4V^2$ ,  $uv_{,m}$  is not real, and in fact we are always in the case of Section 4.5.2, where the coefficients tend to zero. Thus we have non-zero terms only when  $m \geq 4V^2$ .

Writing  $a^{(l)} = -r(\gamma + \sqrt{l\alpha}) = -rC/\sqrt{l\alpha}$ , and plugging (4.20) into (4.13), we get

$$\begin{aligned} \frac{dP^{(l)}}{dz} a^{(l)} &\approx - \int_{uv_{,m}}^{1-uv_{,m}} F^{-1}(u) rC \left( \frac{1}{m} - \frac{1}{u} \right) g^{(l)}(u, \sqrt{\alpha l}z + \alpha l, m) du, \\ &\rightarrow -\frac{rC}{m} P^{(l)} + rCK_1^{(m)}, \end{aligned} \quad (4.22)$$

as  $l \rightarrow \infty$ , where

$$\begin{aligned} K_1^{(m)} &\triangleq \int_{uv_{,m}}^{1-uv_{,m}} F^{-1}(u) \frac{1}{u} \frac{m}{u} \frac{1}{\sqrt{2\pi mu(1-u)}} \exp \left\{ \frac{-(1-u)m}{2u} \right\} du \\ &= \int_{w_0^{(m)}}^{w_1^{(m)}} F^{-1}\left(\frac{m}{m+w^2}\right) \frac{w^2+m}{m} \sqrt{\frac{2}{\pi}} e^{-w^2/2} dw \end{aligned} \quad (4.23)$$

where the second line follows from a change of variables  $w = \sqrt{(1-u)m/u}$ ,

$$w_1^{(m)} \triangleq \sqrt{(1-uv_{,m})m/uv_{,m}},$$

and

$$w_0^{(m)} \triangleq \sqrt{uv_{,m}m/(1-uv_{,m})}.$$

Similarly, using  $\sigma^{(l)2} = r(1 + \gamma/\sqrt{l\alpha}) = rC/(l\alpha)$ , and plugging (4.21) into (4.14) leads to

$$\begin{aligned} \frac{d^2 P^{(l)}}{dz^2} \sigma^{(l)2} &\approx \int_{uv_{,m}}^{1-uv_{,m}} F^{-1}(u) rC \left( \frac{-2}{Bu} + \frac{1}{u^2} - \frac{1}{u(1-u)B} \right) \times \\ &\quad \frac{m}{u} \frac{1}{\sqrt{2\pi mu(1-u)}} \exp \left\{ \frac{-(1-u)m}{2u} \right\} du, \end{aligned}$$

$$\rightarrow rC \left( -2K_1^{(m)}/B + K_2^{(m)} \right) \quad (4.24)$$

as  $l \rightarrow \infty$ , where

$$\begin{aligned} K_2^{(m)} &\triangleq \int_{uv,m}^{1-uv,m} F^{-1}(u) \frac{1}{u^2} \left( 1 - \frac{u}{(1-u)B} \right) \times \\ &\quad \frac{m}{u} \frac{1}{\sqrt{2\pi mu(1-u)}} \exp \left\{ \frac{-(1-u)m}{2u} \right\} du \\ &= \int_{w_0^{(m)}}^{w_1^{(m)}} F^{-1}\left(\frac{m}{m+w^2}\right) \left( \frac{w^2+m}{m} \right)^2 \left( 1 - \frac{1}{w^2} \right) \sqrt{\frac{2}{\pi}} e^{-w^2/2} dw. \end{aligned} \quad (4.25)$$

Finally,

$$\begin{aligned} \frac{dP^{(l)}}{dz} \sigma^{(l)} &\approx \int_{uv,m}^{1-uv,m} F^{-1}(u) \sqrt{rC} \left( \frac{1}{B} - \frac{1}{u} \right) g^{(l)}(u, \sqrt{\alpha l}z + \alpha l, B) du, \\ &\rightarrow \frac{\sqrt{rC}}{B} P^{(l)} - \sqrt{rC} K_1^{(m)}. \end{aligned}$$

Substituting all of the above into (4.12) yields (4.15).

#### 4.6.1 Example: $F$ uniform

Suppose  $F$  is the uniform distribution on  $[0, \bar{P}]$ . Then  $F^{-1}(\frac{m}{m+w^2}) = \bar{P} \frac{m}{m+w^2}$ , hence

$$\begin{aligned} K_1^{(m)} &= \bar{P} \int_{w_0^{(m)}}^{w_1^{(m)}} \sqrt{\frac{2}{\pi}} e^{-w^2/2} dw \\ &= 2\bar{P} [\Phi(w_1^{(m)}) - \Phi(w_0^{(m)})], \end{aligned}$$

where  $\Phi$  is the normal distribution of mean zero, and variance 1. Also,

$$\begin{aligned} K_2^{(m)} &= \bar{P} \int_{w_0^{(m)}}^{w_1^{(m)}} (w^2/m + 1 - 1/m - 1/w^2) \sqrt{\frac{2}{\pi}} e^{-w^2/2} dw \\ &= \bar{P} \sqrt{\frac{2}{\pi}} \left[ e^{-w_0^{(m)2}/2} \left( \frac{w_0^{(m)}}{m} - \frac{1}{w_0^{(m)}} \right) - e^{-w_1^{(m)2}/2} \left( \frac{w_1^{(m)}}{m} - \frac{1}{w_1^{(m)}} \right) \right] + 2K_1^{(m)}. \end{aligned}$$

#### 4.7. Optimal Seller Strategies

In this section, by way of conclusion, we briefly outline approaches to the complementary control problem, i.e., how the seller can “tune” the mechanism in order to optimize some system-wide objective.

Taking the above mechanism as a given, consider the optimal strategy for the network manager, whose objective is to maximize revenue, using  $\mu(P) \in [0, \infty]$  as the (feedback) control. As the spot market is governed by auctions,  $P \equiv 0$  whenever the system is not full. The relevance of the problem is during periods when the system is heavily loaded (i.e., almost always full). Then, the objective is equivalent to minimizing

$$J(s, y, \mu) = - \int_s^\tau P_t dt,$$

over all admissible controls  $\mu(t) \equiv \mu[P]$ .  $\tau$  is the stopping time  $\tau = \inf\{t > s : P_t = 0\}$ , and the initial state is  $P_s = y$ . The dynamic programming equation is

$$0 = \frac{\partial W}{\partial s} + \min_\mu (\mathcal{A}W - y),$$

for  $y > 0$ , and  $W(s, 0) = 0$ . The differential operator  $\mathcal{A}$  is given by [20]

$$\mathcal{A} = \frac{1}{2} [D(\mu)y + K_2(\mu)] \frac{\partial^2}{\partial y^2} + [A(\mu)y + K_1(\mu)] \frac{\partial}{\partial y}.$$

If  $W$  is a solution of the above equation, then the optimal cost is  $J(s, y, \mu^*) = W(s, y)$ , and, in the classical approaches, the optimal control is obtained by performing the minimization over  $\mu$ . In this case, even when  $W$  can be solved for, or guessed, for each  $y$  finding the control  $\mu(y)$  is in general not tractable since it involves polynomials of degree  $B!$ .

There are a number of promising approaches to this control problem. It is inter-

esting to investigate the applicability of approximation techniques by discretizing the original problem, and solving it as a Markov-decision problem, in the manner of [53] as well as that of [81], and compare the resulting performance with the optimal open-loop control. One should also consider some heuristic solutions based on related but simpler solvable problems, and investigate the relationship of the optimal control with the notion of Brownian local time, as in [11]. Finally, the trade-offs between optimality and computational complexity should be investigated.

What is claimed is:

1. A method for real-time market-based resource allocation, comprising:  
 receiving at least one bid in real-time for a resource;  
 deciding which of the at least one bids has won the bidding; and  
 controlling the resource so that it is committed to the winning bidder.
2. The method of claim 1, wherein the resource is bandwidth.
3. The method of claim 1, wherein the resource is buffer space.
4. The method of claim 1, wherein the resource is processor time.
5. The method of claim 1, wherein the resource is controlled in real-time so that the winning bidder has access to the resource he has won as soon as bidding closes.
6. The method of claim 1, wherein the bid is received from a software agent.
7. The method of claim 6, wherein the software agent bids in accordance with a strategy rule.
8. The method of claim 7, wherein the strategy rule is a truthful best reply strategy.
9. The method of claim 6, wherein the software agent bids in accordance with a valuation rule.
10. The method of claim 9, wherein the valuation rule is determined in accordance with at least one measured network parameter.
11. The method of claim 6, wherein the software agent bids in accordance with an allocation rule.
12. The method of claim 1, wherein deciding which bid has won the bidding is performed in accordance with an allocation rule.
13. The method of claim 12, wherein deciding which bid has won the bidding is performed in accordance with a market allocation rule also used by a buyer software agent, the market allocation rule defining the rules of the resource market.
14. The method of claim 12, wherein deciding which bid has won the bidding is performed in accordance with an English Auction market allocation rule.
15. The method of claim 12, wherein deciding which bid has won the bidding is performed in accordance with a continuous bid-ask trading market allocation rule.
16. The method of claim 12, where deciding which bid has won the bidding is performed in accordance with a progressive second price auction allocation rule.
17. The method of claim 12, where deciding which bid has won the bidding is performed in accordance with a hold option allocation rule.
18. The method of claim 1, further comprising:  
 storing information concerning which bid has won, for accounting purposes.
19. The method of claim 1, wherein there are several resources, and a respective resource agent performs the elements of claim A for each resource.
20. The method of claim 1, wherein there is one resource, and a single resource agent performs the elements of claim 1 for the resource.
21. The method of claim 1, wherein the elements of claim 1 are performed by a resource agent executing on a separate computer than the placing the bids.
22. The method of claim 1, wherein the elements of claim 1 are performed by a resource agent executing on the same computer as at least one placing at least one of the bids.
23. The method of claim 1, wherein it is controlled by a human being, who decides how to bid.
24. The method of claim 1, further comprising:  
 deciding, by a bidder agent, how to bid based on a valuation function of the bidder agent and a strategy algorithm of the.
25. The method of claim 1, further comprising: receiving an offer to sell resources from a seller agent.
26. The method of claim 1, further comprising:  
 receiving player agents in a garage of a resource agent and receiving bids from player agents in the garage.
27. The method of claim 1, wherein the bid is received from a player agent that also submits offers to sell resources.
28. The method of claim 1, further comprising:  
 before bidding, receiving a request from a buyer software agent for a location of resource agent.
29. The method of claim 1, further comprising:  
 sending at least one bid in real-time for a resource;  
 receiving a notification that the sent bid has won the bidding;  
 selling the use of the winning resource to third parties through a separate resource agent; and  
 notifying a resource management and control agent that the third parties will be using the resource.
30. A method, performed by a multiagent, comprising:  
 sending at least one bid in real-time for a resource in accordance with a strategy rule and a valuation rule of the multiagent and in accordance with a system-wide allocation rule;  
 receiving a notification that the sent bid has won the bidding;  
 selling the use of the winning resource to third parties through a separate resource agent; and  
 notifying a resource management and control agent that the third parties will be using the resource.
31. A method, performed by a buyer agent, comprising:  
 sending at least one bid in real-time for a resource in accordance with a strategy rule and a valuation rule of the buyer agent and in accordance with a system-wide allocation rule;  
 receiving a notification that the sent bid has won the bidding; and  
 making use of the resource in real time, immediately after winning the bid.
32. The method of claim 31, wherein the system wide-allocation rule is a PSP allocation rule.
33. A system for real-time market-based resource allocation, comprising:  
 means for receiving at least one bid in real-time for a resource;  
 means for deciding which of the at least one bids has won the bidding; and

means for controlling the resource so that it is committed to the winning bidder.

**34. A system for real-time market-based resource bidding by a multiagent, comprising:**

means for sending at least one bid in real-time for a resource in accordance with a strategy rule and a valuation rule of the multiagent and in accordance with a system-wide allocation rule;

means for receiving a notification that the sent bid has won the bidding;

means for selling the use of the winning resource to third parties through a separate resource agent; and

means for notifying a resource management and control agent that the third parties will be using the resource.

**35. A system for real-time market-based resource bidding by a buyer agent, comprising:**

means for sending at least one bid in real-time for a resource in accordance with a strategy rule and a valuation rule of the buyer agent and in accordance with a system-wide allocation rule;

means for receiving a notification that the sent bid has won the bidding; and

means for making use of the resource in real time, immediately after winning the bid.

**36. The system of claim 35, wherein the system wide-allocation rule is a PSP allocation rule.**

**37. A system for real-time market-based resource allocation, comprising:**

a portion configured to receive at least one bid in real-time for a resource;

a portion configured to decide which of the at least one bids has won the bidding; and

a portion configured to control the resource so that it is committed to the winning bidder.

**38. A system for real-time market-based resource bidding by a multiagent, comprising:**

a portion configured to send at least one bid in real-time for a resource in accordance with a strategy rule and a valuation rule of the multiagent and in accordance with a system-wide allocation rule;

a portion configured to receive a notification that the sent bid has won the bidding;

a portion configured to sell the use of the winning resource to third parties through a separate resource agent; and

a portion configured to notify a resource management and control agent that the third parties will be using the resource.

**39. A system for real-time market-based resource bidding by a buyer agent, comprising:**

a portion configured to send at least one bid in real-time for a resource in accordance with a strategy rule and a valuation rule of the buyer agent and in accordance with a system-wide allocation rule;

a portion configured to receive a notification that the sent bid has won the bidding; and

a portion configured to make use of the resource in real time, immediately after winning the bid.

**40. The system of claim 39, wherein the system wide-allocation rule is a PSP allocation rule.**

**41. A computer program product, including instructions for causing a data processing device to perform actions for real-time market-based resource allocation, comprising:**

receiving at least one bid in real-time for a resource;

deciding which of the at least one bids has won the bidding; and

controlling the resource so that it is committed to the winning bidder.

**42. A computer program product, including instructions for causing a multiagent of a data processing device to perform actions for real-time market-based resource bidding, comprising:**

sending at least one bid in real-time for a resource in accordance with a strategy rule and a valuation rule of the multiagent and in accordance with a system-wide allocation rule;

receiving a notification that the sent bid has won the bidding;

selling the use of the winning resource to third parties through a separate resource agent; and

notifying a resource management and control agent that the third parties will be using the resource.

**43. A computer program product, including instructions for causing a buyer agent of a data processing device to perform action for real-time market-based resource bidding, comprising:**

sending at least one bid in real-time for a resource in accordance with a strategy rule and a valuation rule of the buyer agent and in accordance with a system-wide allocation rule;

receiving a notification that the sent bid has won the bidding; and

making use of the resource in real time, immediately after winning the bid.

**44. The computer program product of claim 43, wherein the system wide-allocation rule is a PSP allocation rule.**

\* \* \* \* \*